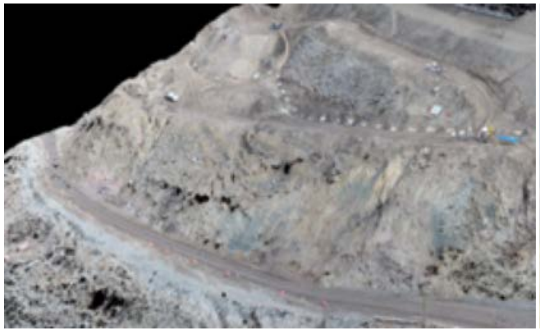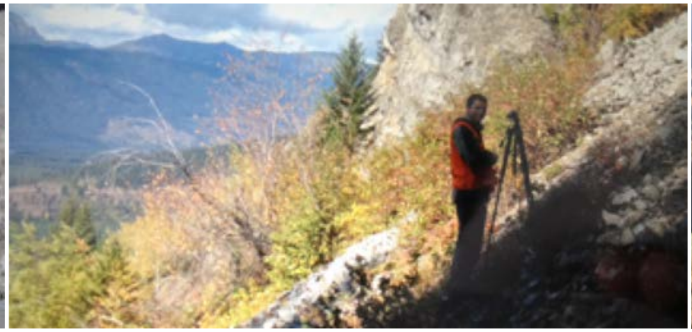

Uncertainty and Probability

in rock slope stability assessment and design

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Introduction

A through investigation of slope design criteria in open pit mining led Wesseloo and Read (in: Read and Stacey, 2009) to develop the following summary:

Slope scale	Failure consequence	Acceptance criteria		
		FoS (min) (static)	FoS (min) (Dynamic)	PoF (max) [FoS≤1]
Bench	Low-high	1.1	NA	20 – 50%
Inter-ramp	Low	1.15 - 1.2	1.0	25%
	Medium	1.2	1.0	20%
	High	1.2 – 1.3	1.1	10%
Overall	Low	1.2 – 1.3	1.0	15 – 20%
	Medium	1.3	1.05	5 – 10%
	High	1.3 – 1.5	1.1	≤ 5%

- It is intuitive that FoS increase (PoF decrease) for larger slopes
- But we also know: as long as strength is higher than stresses -> OK
- 1) How is this consistent? 2) What is the role of performance?

Introduction

Two important considerations to answer the first question:

- Our "**confidence**" on our assessment of the slope -> Failure likelihood
- How much it will "**hurt**" given the slope fails -> Failure consequence

Therefore the criteria, as we understand it now, is a form of **risk assessment**.

Performance becomes a means of validating/calibrating our understanding of the slope behavior and therefore allows us to become more "confident" of our assessment of the slope. This gives room for increasing the efficiency of slope design (lower FoS or constant PoF for steeper slopes).

<u>Key points</u>:

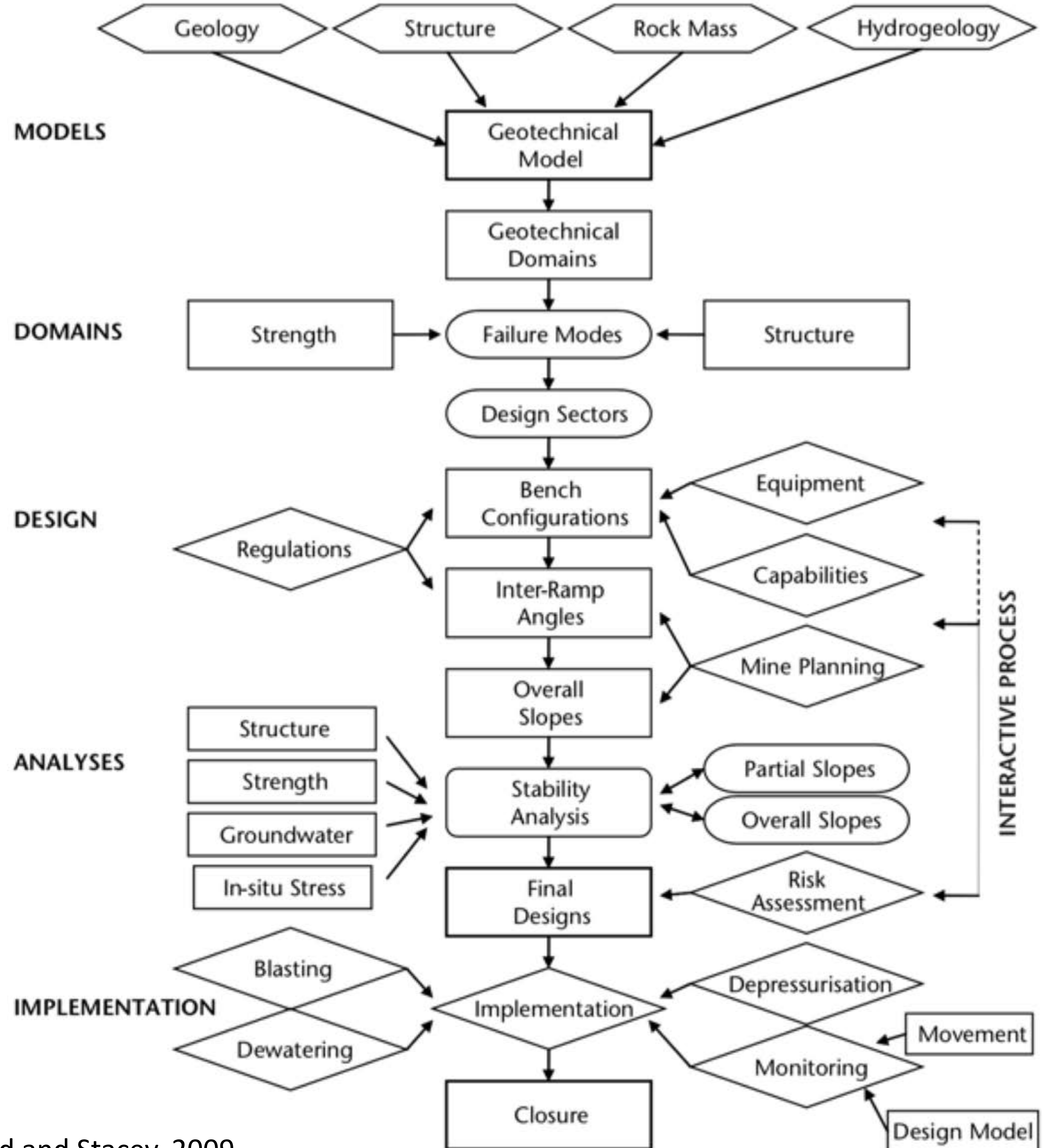
- The level of confidence on our slope assessments is driven by the **uncertainty** in our analyses.
- There are different approaches to manage uncertainty, which will reflect on the criteria selected for slope evaluation

Today:

- Sources of uncertainty in rock slope engineering
  - Model uncertainty
  - Parameter uncertainty
  - Human uncertainty
- Uncertainty and probability in slope assessments
  - Notions of probability
  - Quantifying parameter uncertainty
  - Propagating uncertainty in slope analyses
  - Reliability, FoS and PoF
- Role of monitoring and performance
- Conclusions

Today:

Keep In mind the life of the pit

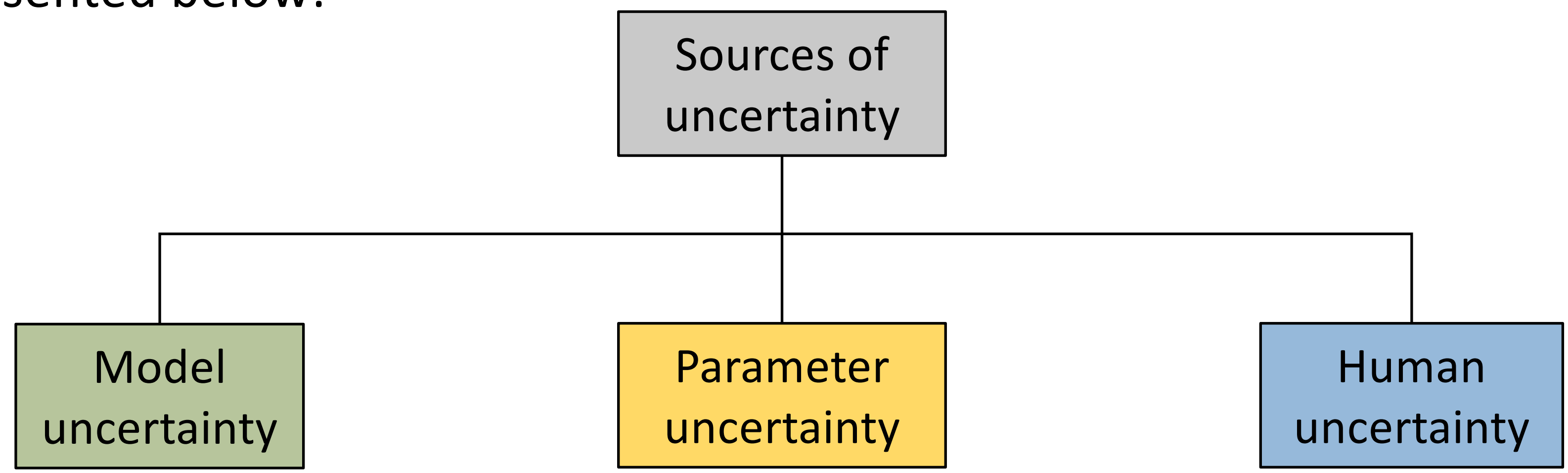


Read and Stacey, 2009

Sources of uncertainty in rock slope engineering

Sources of uncertainty

- There are a number of classifications of uncertainty (or classification of sources of uncertainty) in the literature.
- One classification that proves adequate for the geotechnical practitioner is presented below:



- This classification becomes clear for the geotechnical practitioner as it applies to the analysis of slopes. It addresses the potential for slope failure.
- What about the consequences?

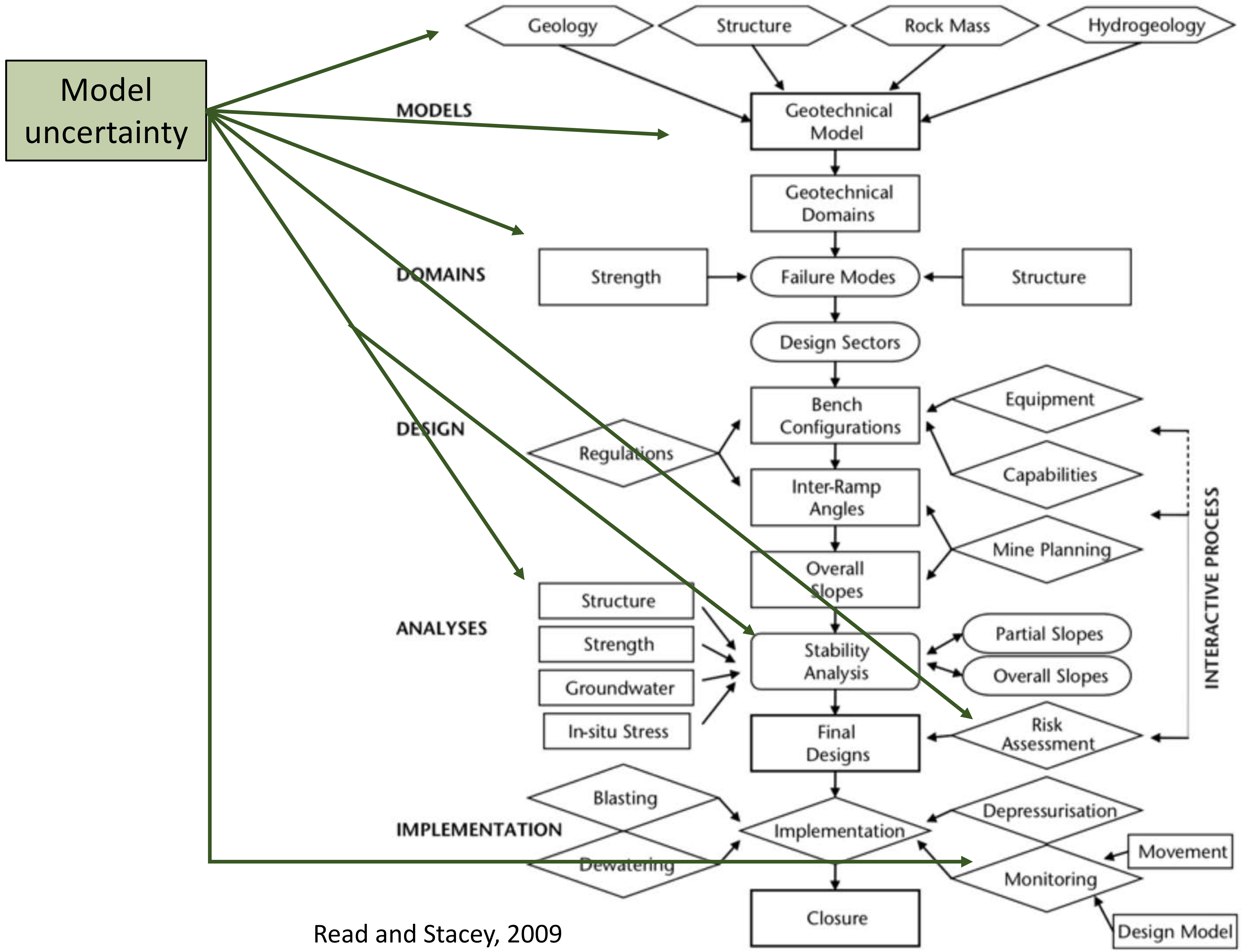
Sources of uncertainty in rock slope engineering

Model uncertainty

- We can discuss two types of models
  - 1) Our mental models of reality (theory)
  - 2) Models for analysis (calculation tools)

- The questions associated with these sources of uncertainty include:
  - Is this a valid theory for the situation?
  - What behavior should I expect during excavation? After?
  - Are the simplifications adequate for analysis? Do we capture the expected behavior?

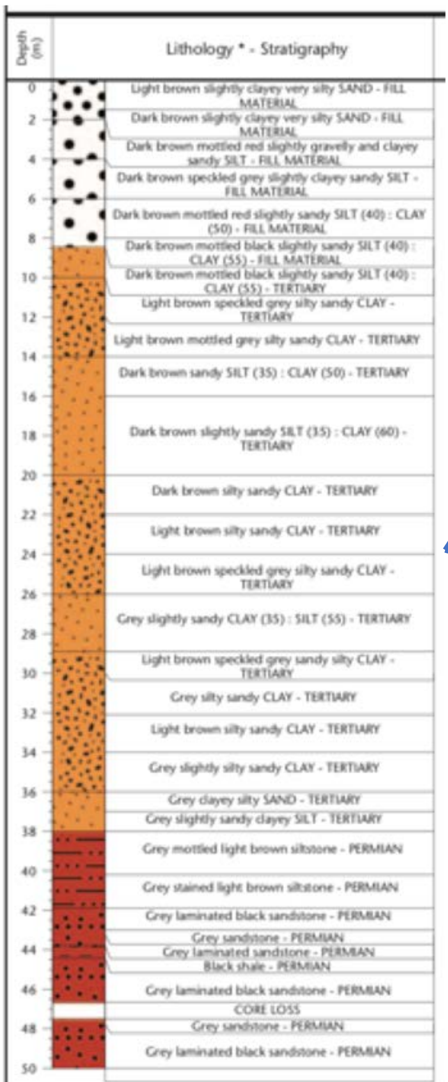
Sources of uncertainty in rock slope engineering



Read and Stacey, 2009

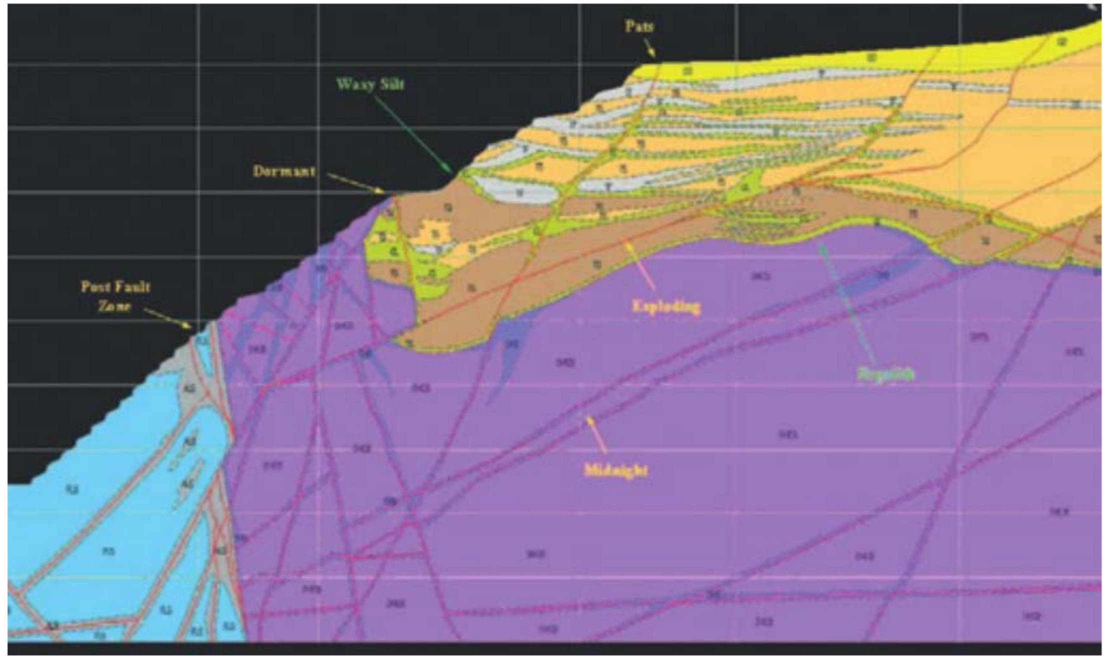
Sources of uncertainty in rock slope engineering

Model uncertainty

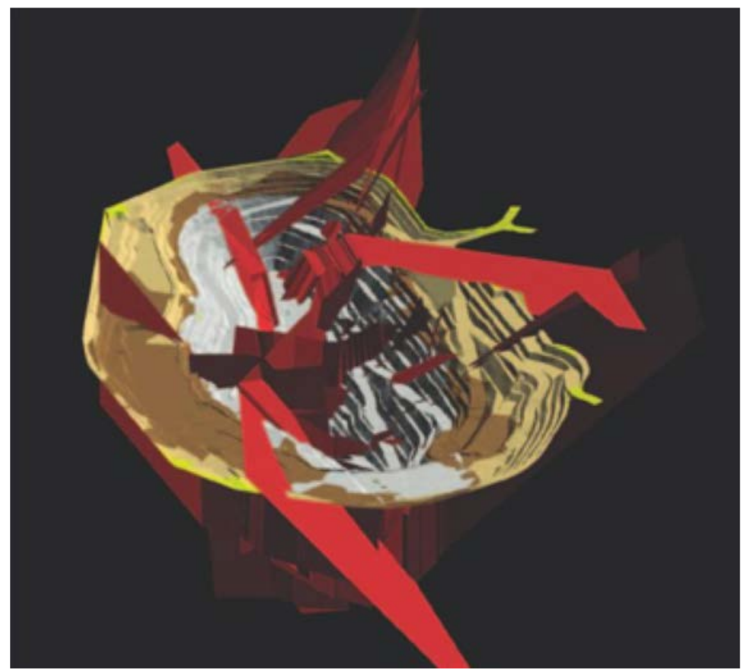


Martin and Stacey, 2017

Investigation data



Geological models

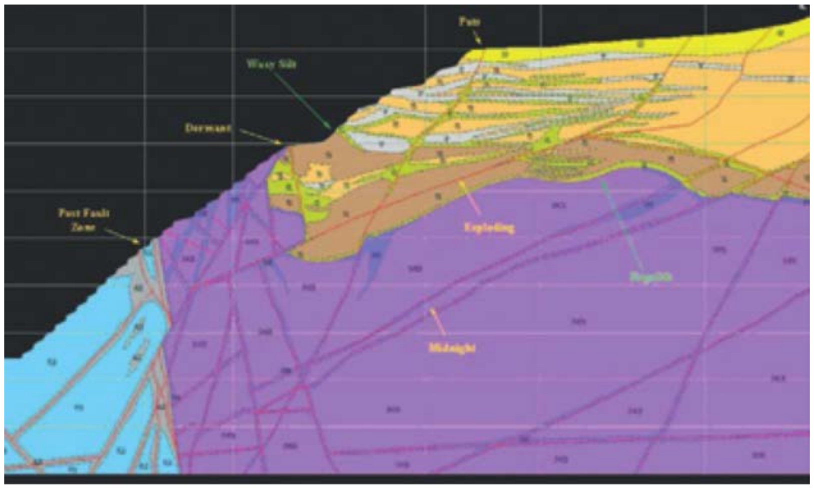


Structural models

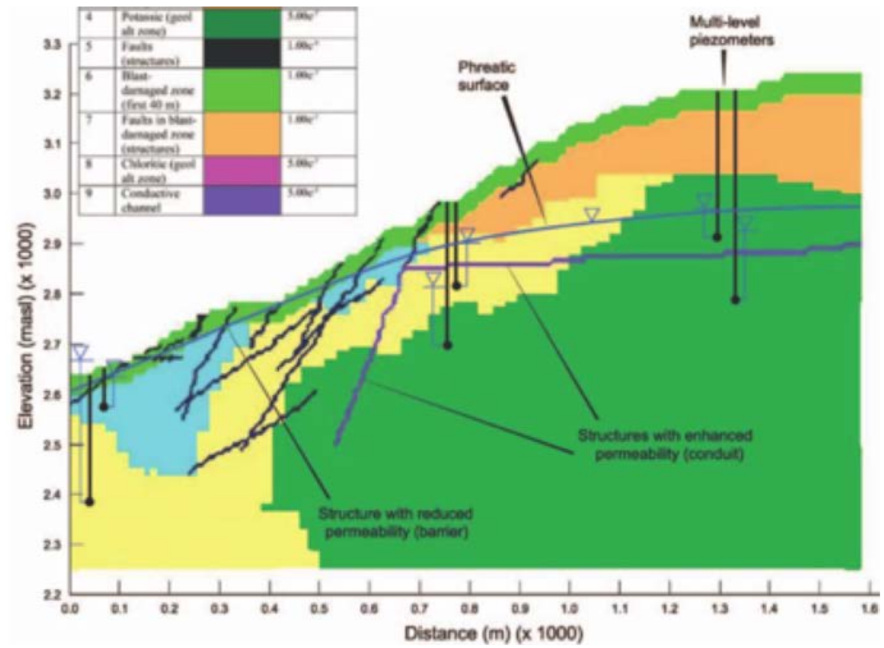
Read and Stacey, 2009

Sources of uncertainty in rock slope engineering

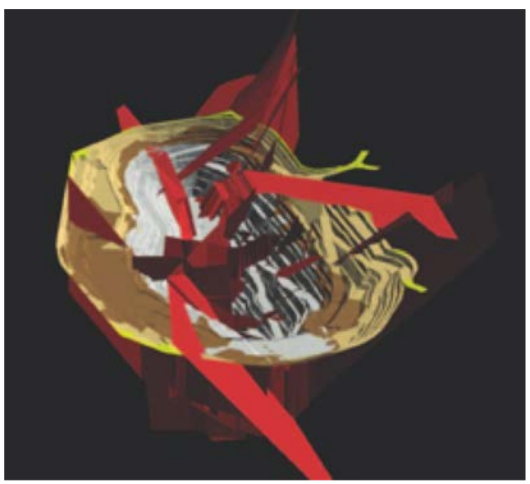
Model uncertainty



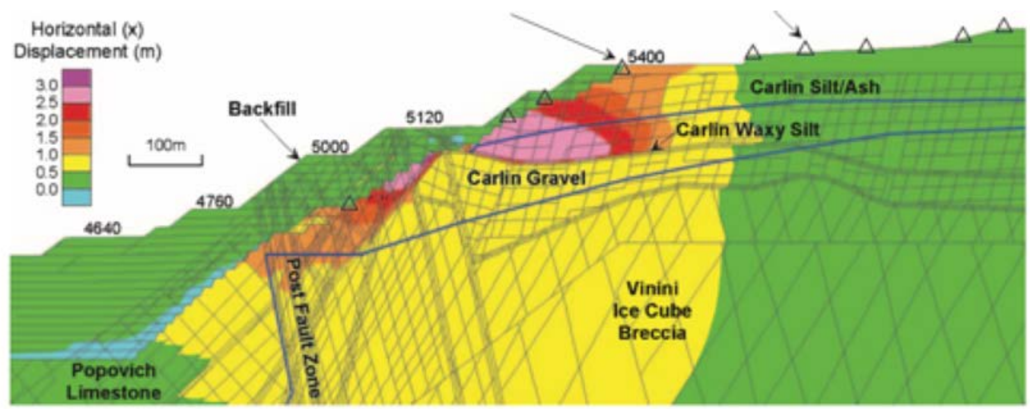
Geological models



Hydrogeological models



Structural models



Stability models

Read and Stacey, 2009
Martin and Stacey, 2017

Sources of uncertainty in rock slope engineering

Model uncertainty

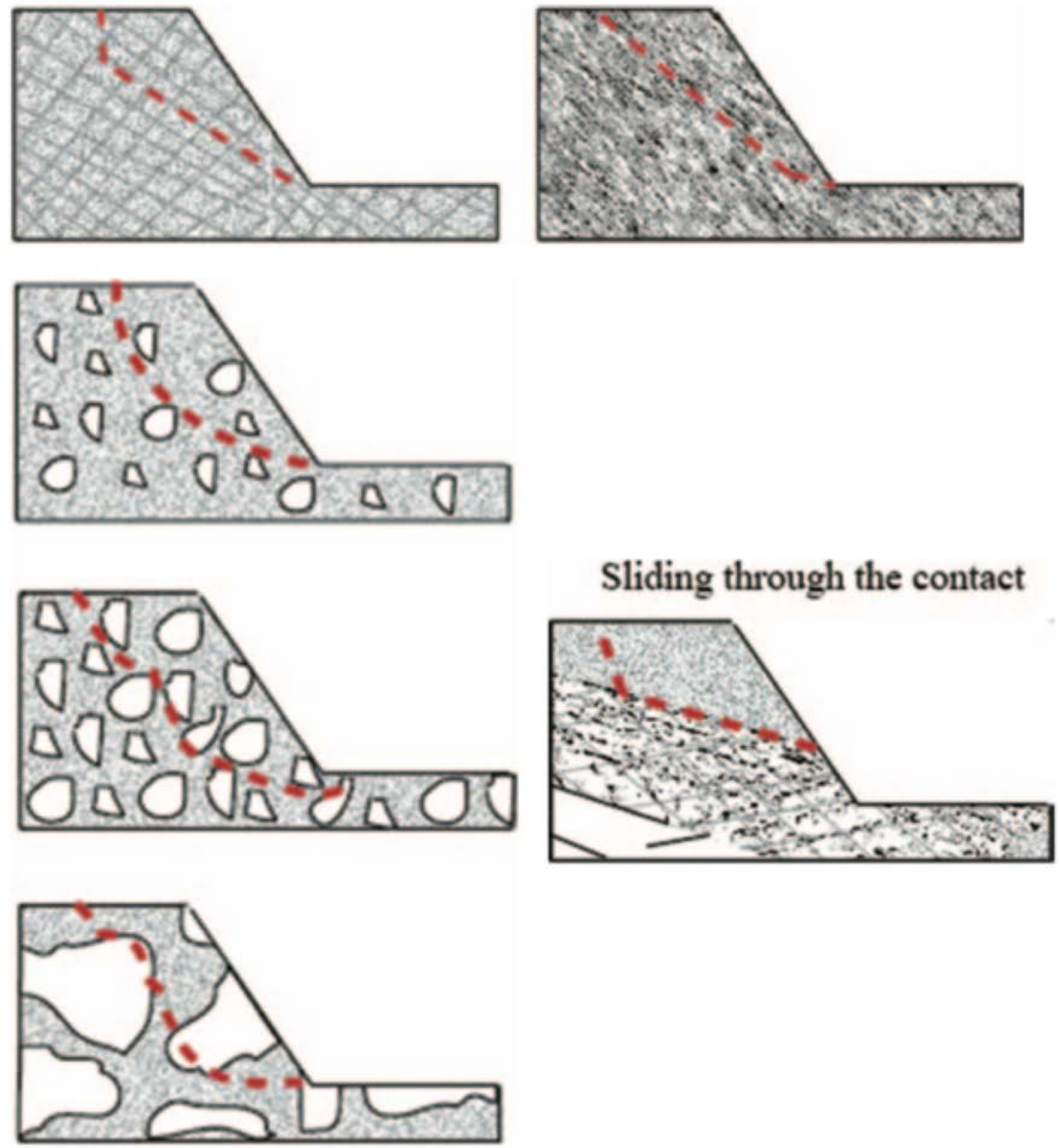
Expected behavior:

- Dilate? Contract?
- Collapsible material? Swelling potential?
- Mohr-Coulomb? Hoek-Brown? Discontinuous?
- Progressive failure?
- Ductile? Brittle?

Tools available for reducing model uncertainty:

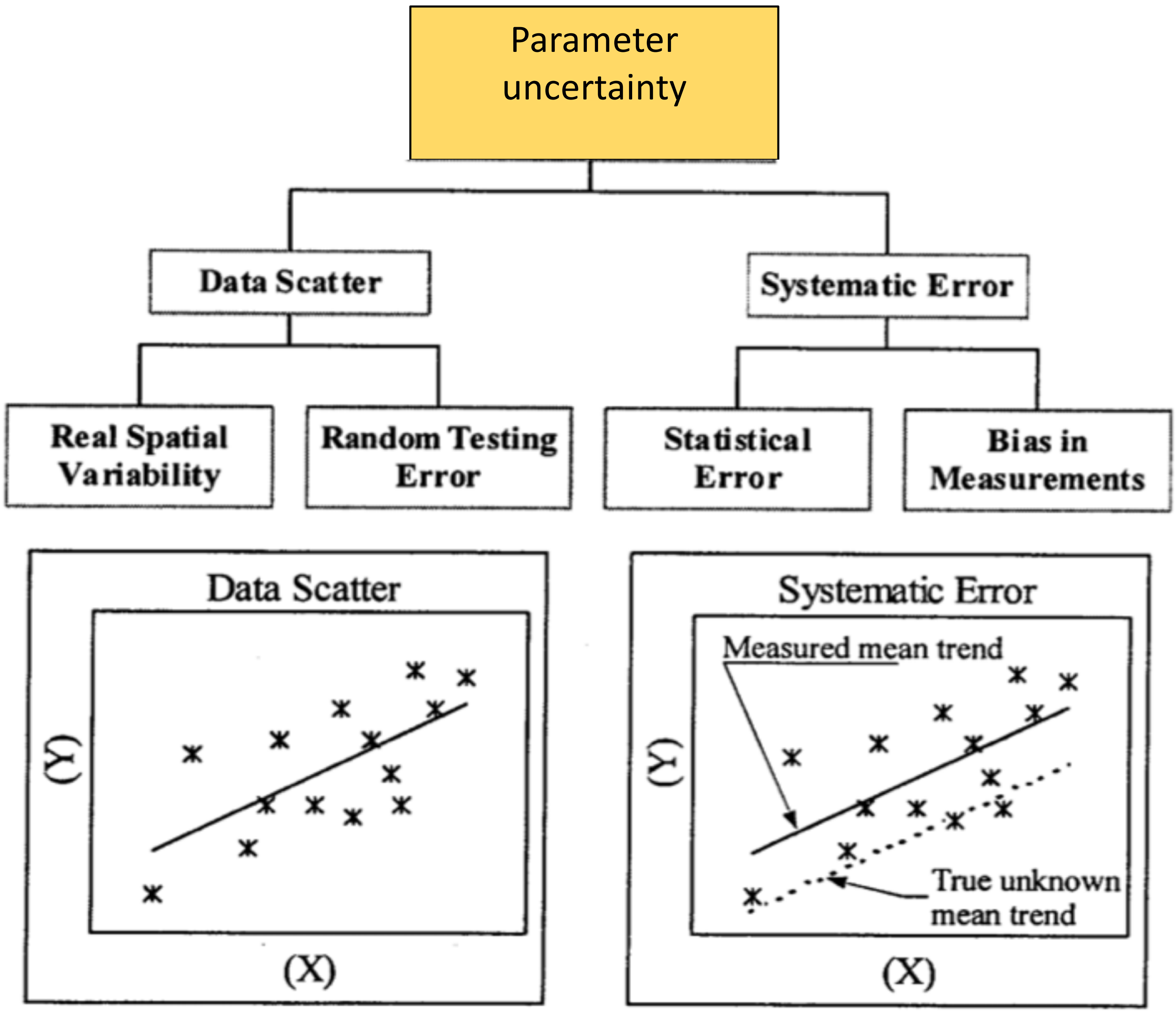
- **Peer reviews**
- **Case history analysis (past performance)**

Expected mode of failure:



Martin and Stacey, 2017

Sources of uncertainty in rock slope engineering



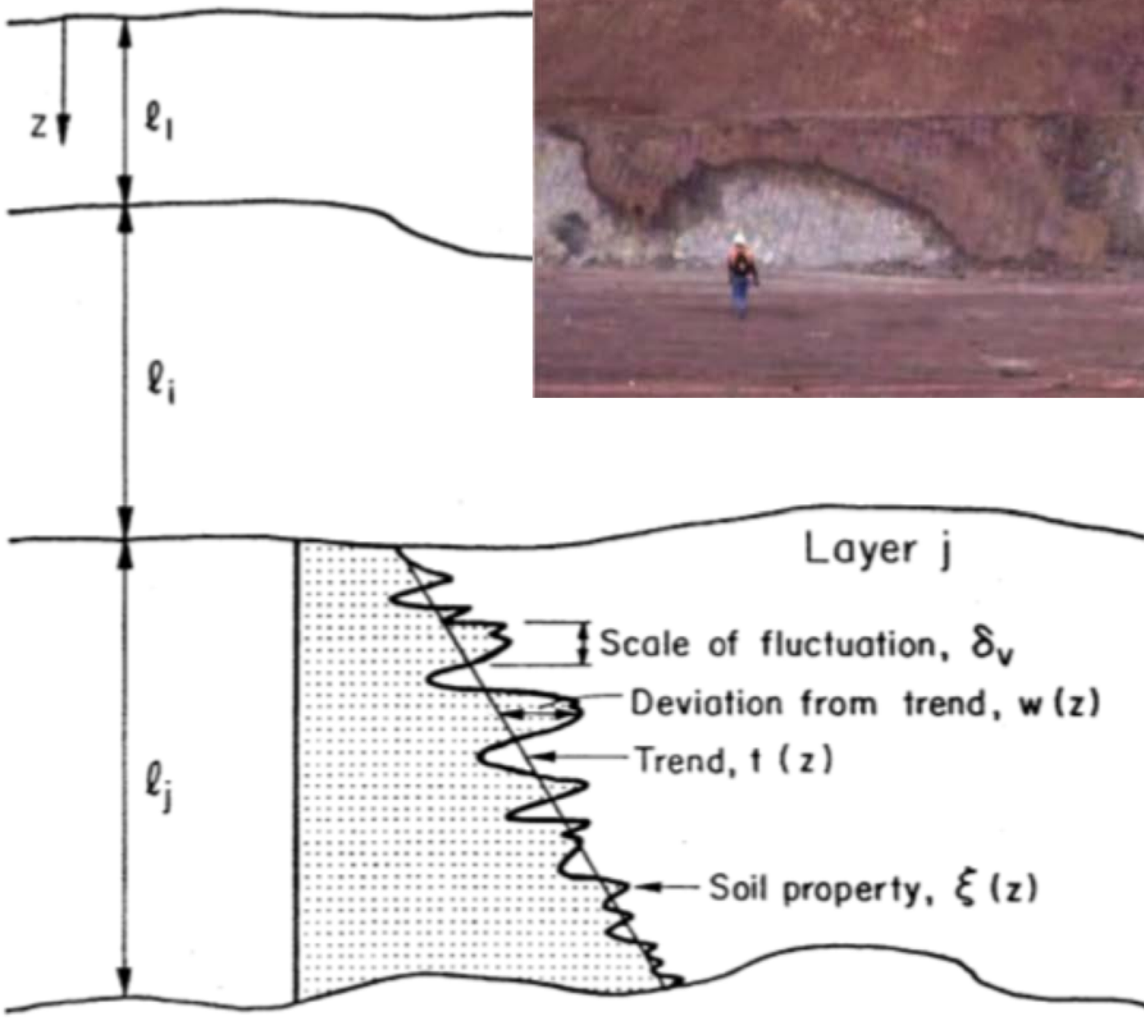
after El-Ramly, 2001

Sources of uncertainty in rock slope engineering

Parameter uncertainty

Spatial variability from heterogeneous weathering

Martin and Stacey, 2017



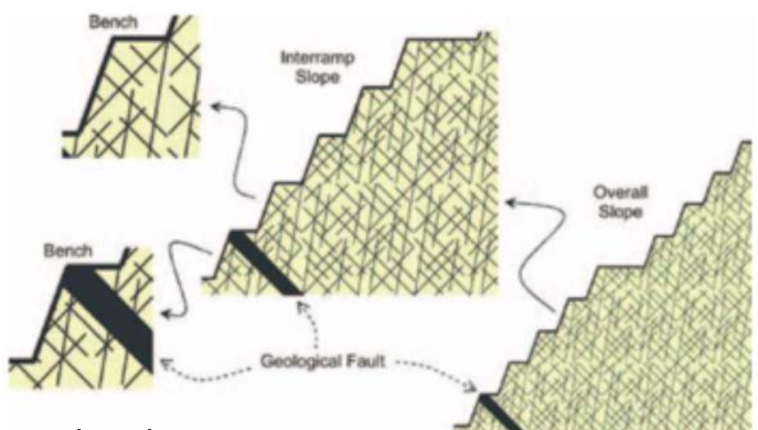
Phoon and Kulhawy, 1999

- Variability with depth.
- Similarly, horizontal variability
- Both depend on the geological history!
- What about in the rock world?

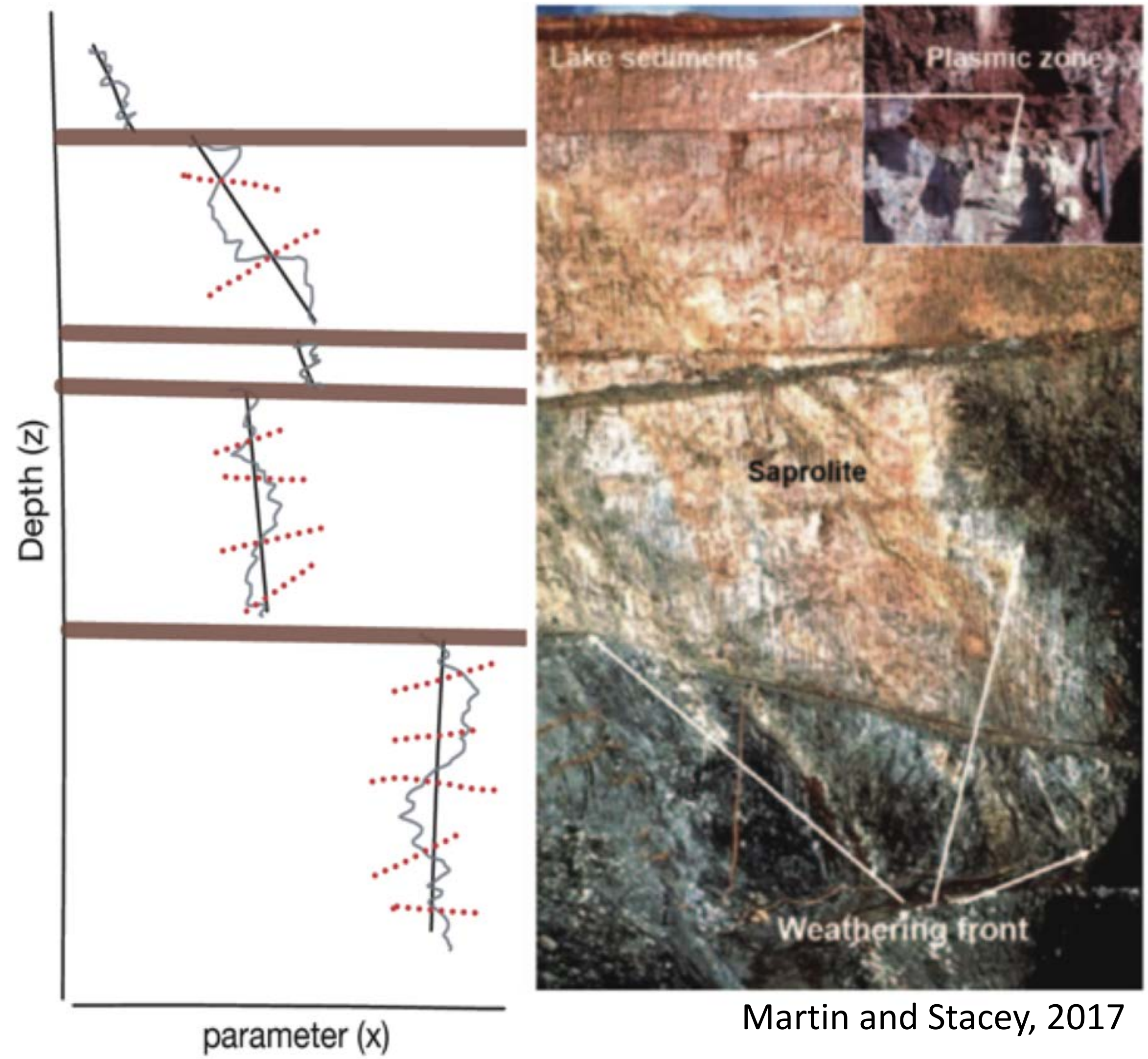
Sources of uncertainty in rock slope engineering

Parameter uncertainty

- Difficulties in the rock world (and some soils) from the complex morphologic history when compared with sediments (alteration, structures, etc.)
- Therefore, Our conceptual models vary with the dimension of the slopes



Read and Stacey, 2009



Martin and Stacey, 2017

Sources of uncertainty in rock slope engineering

Parameter uncertainty

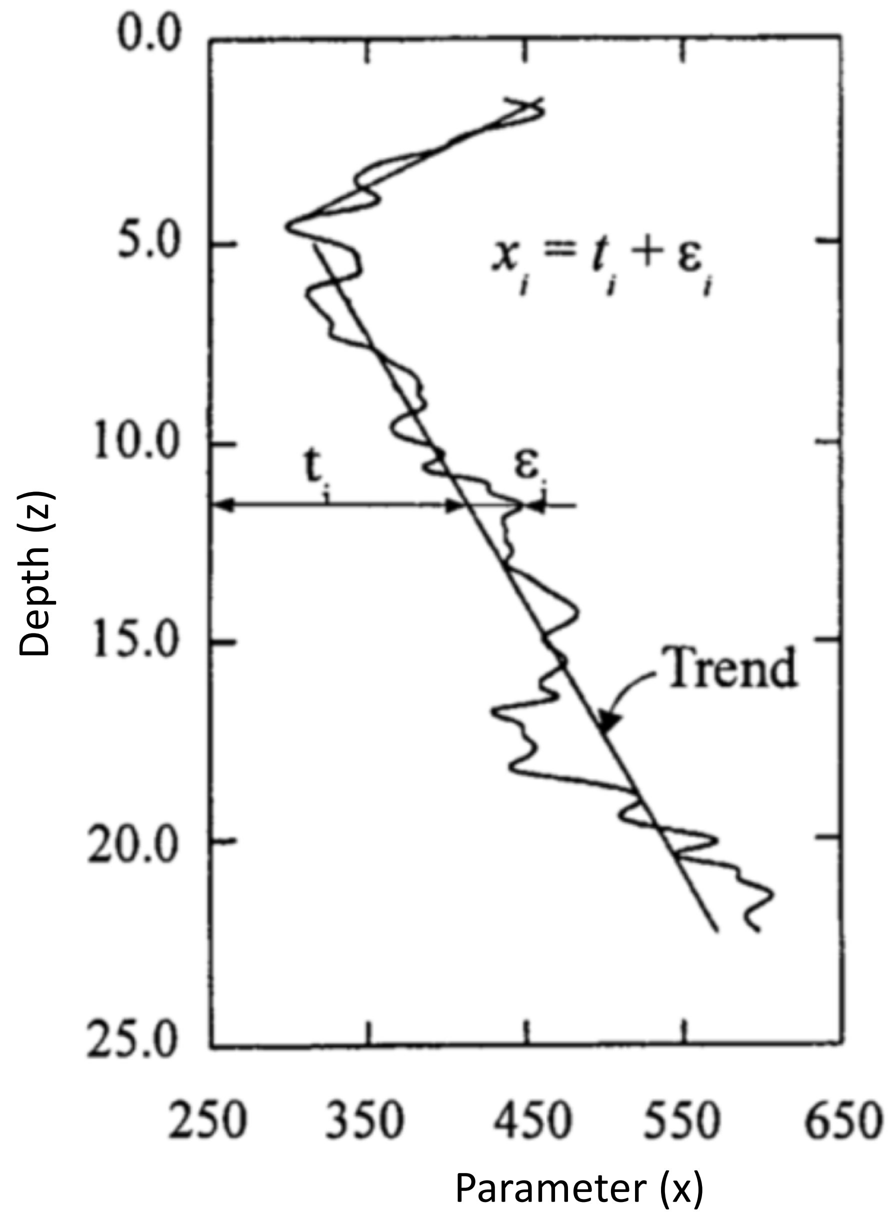
- Multiple approaches with varying degrees of complexity based on non-linearity and multiple parameters
- Basic method is a linear regression:

$$x_i = a_0 + a_i z_i + e_i$$

$$a_0 = E[x] - a_i E[z]$$

$$a_i = \frac{\sum(z_i - E[z])(x_i - E[x])}{\sum(z_i - E[z])^2}$$

$$e_i = ?$$

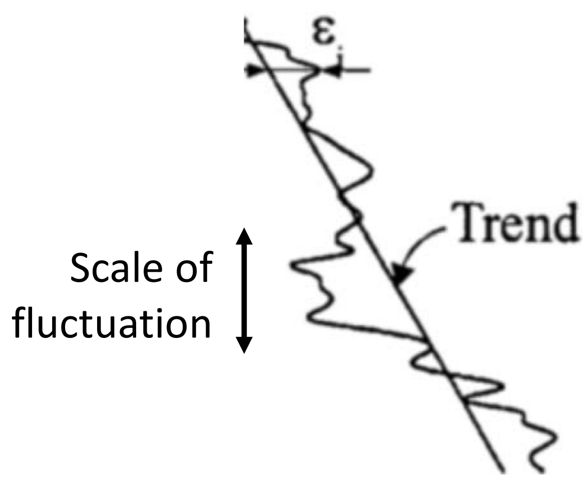


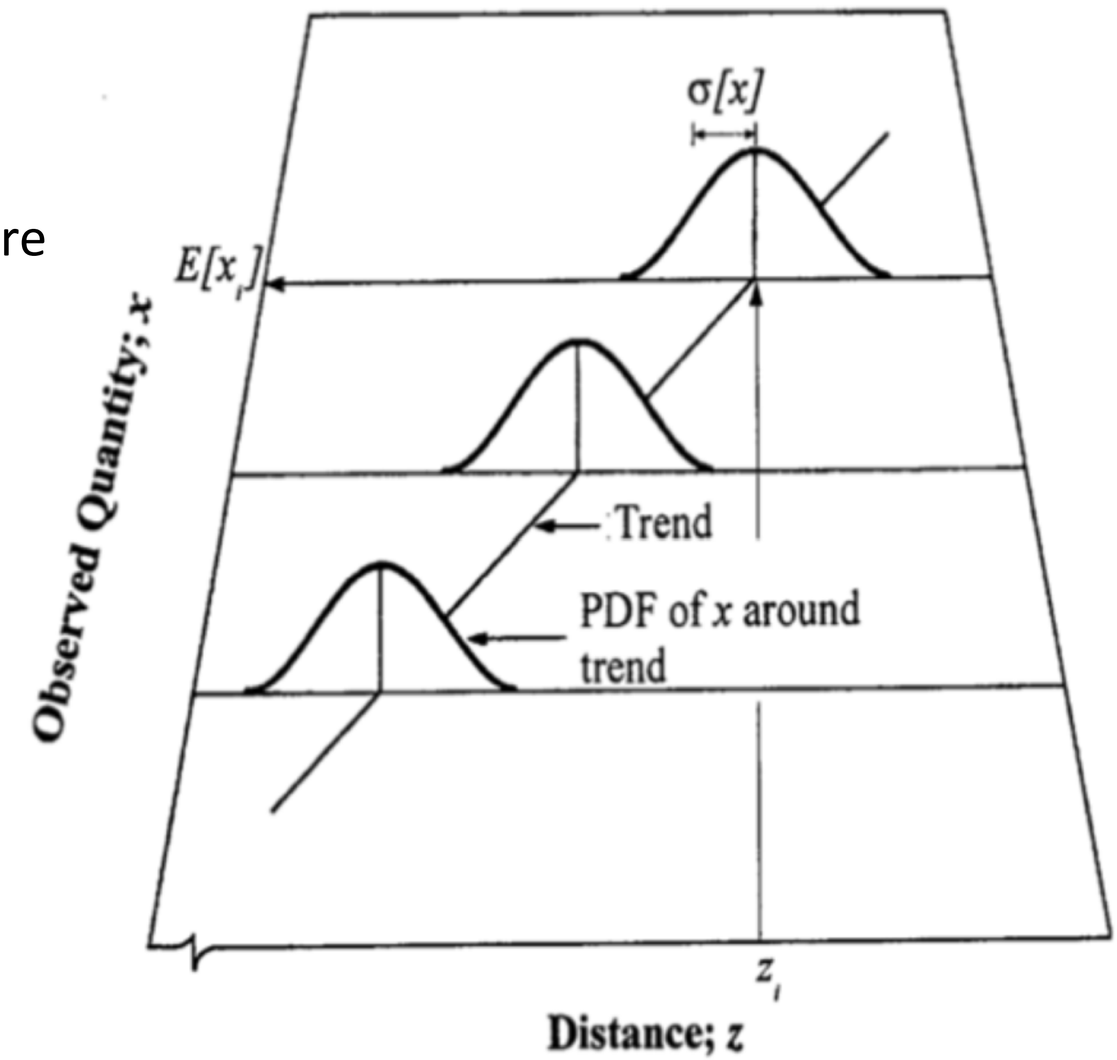
El-Ramly, 2001

Sources of uncertainty in rock slope engineering

Parameter uncertainty

- The trends are considered deterministic, while residuals (e) are treated stochastically.
- Spatial variability then focuses on correlations between residuals.





El-Ramly, 2001

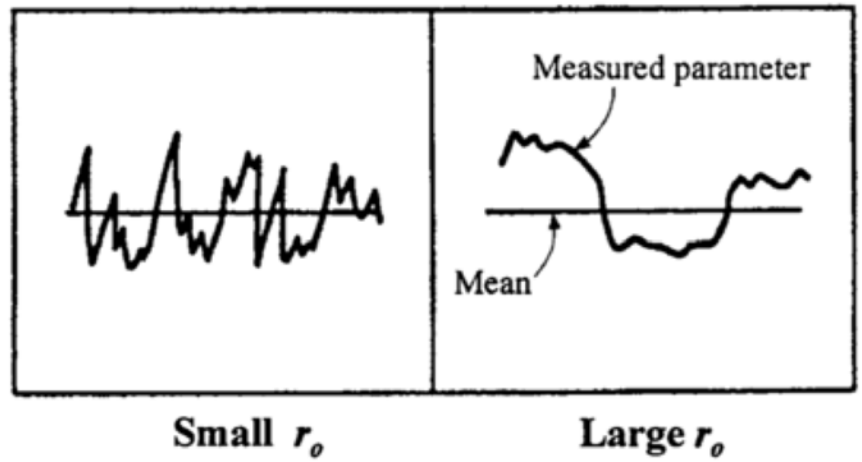
Sources of uncertainty in rock slope engineering

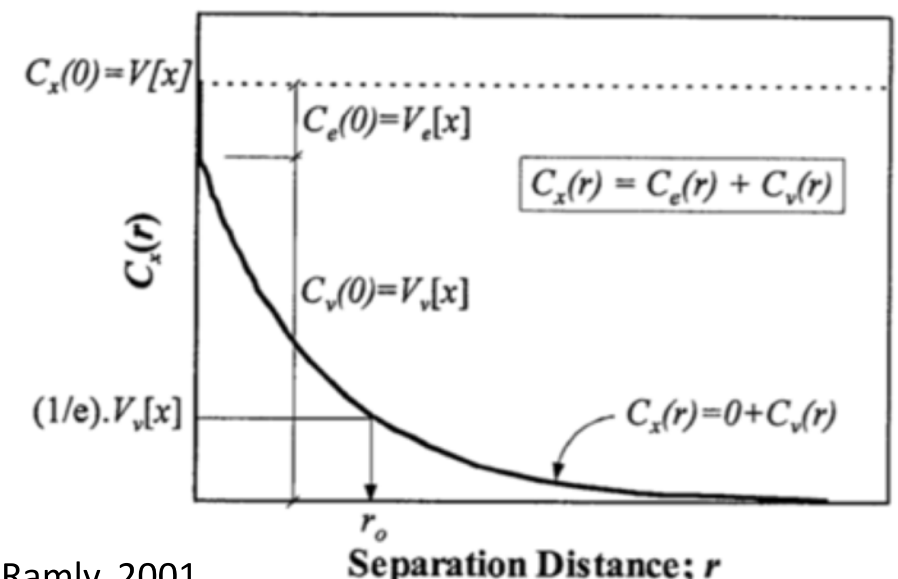
Parameter uncertainty

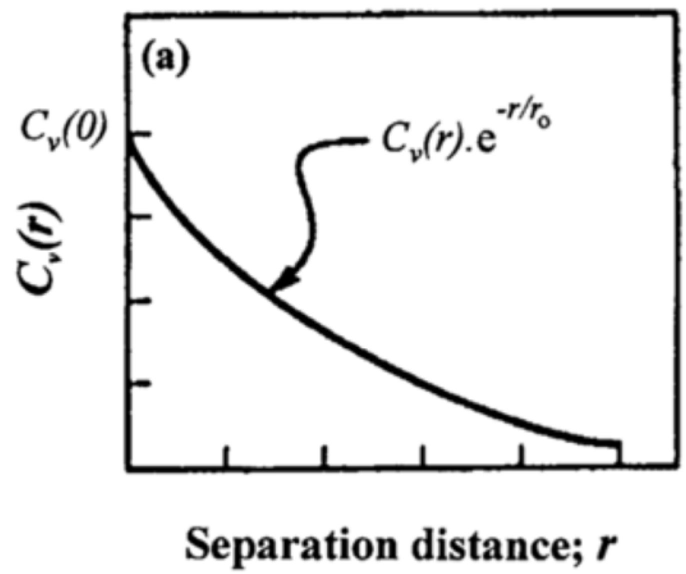
- Autocovariance $C_x(r)$: measure of data (x) "similarity" for a given distance (r)

$$C_x(r) = E[(x_i - t_i)(x_{i+r} - t_{i+r})]$$

- Small r, large $C_x(r)$. Decreases with increasing r.





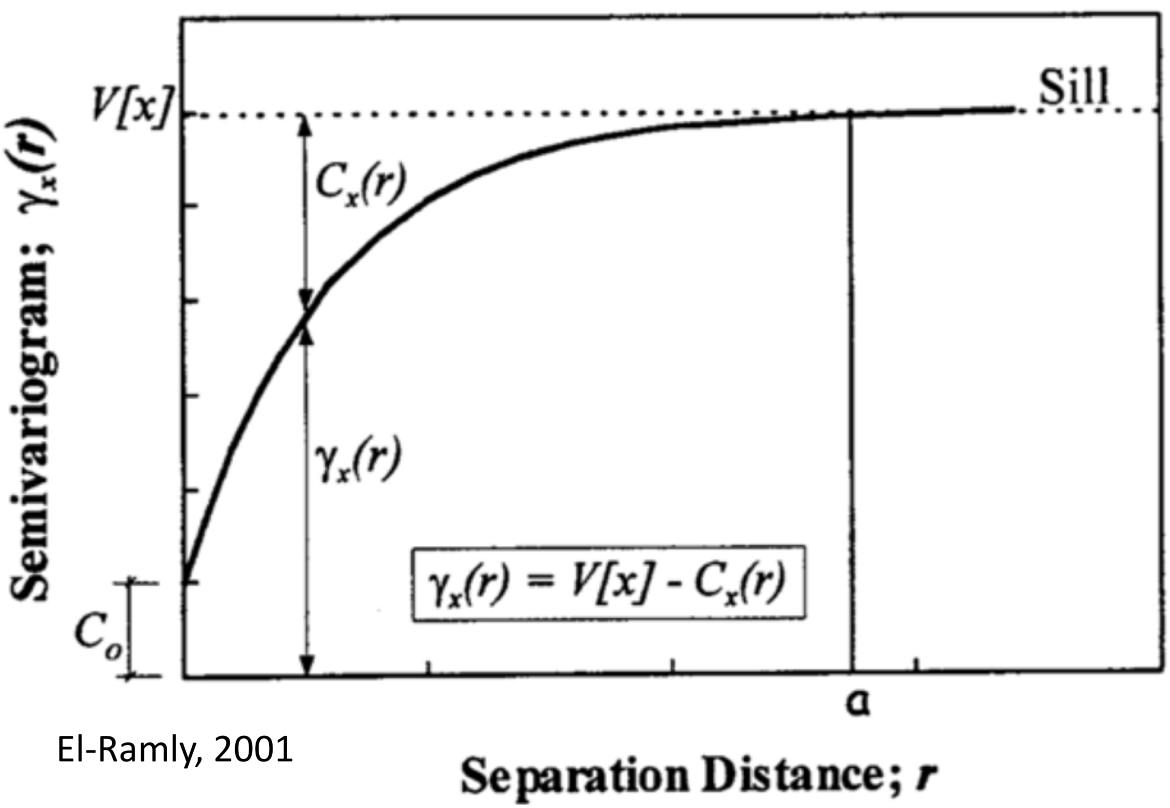


El-Ramly, 2001

Sources of uncertainty in rock slope engineering

Parameter uncertainty

- Semivariograms $\gamma_x(r)$: Commonly used in geostatistics for mining and reservoir characterization.



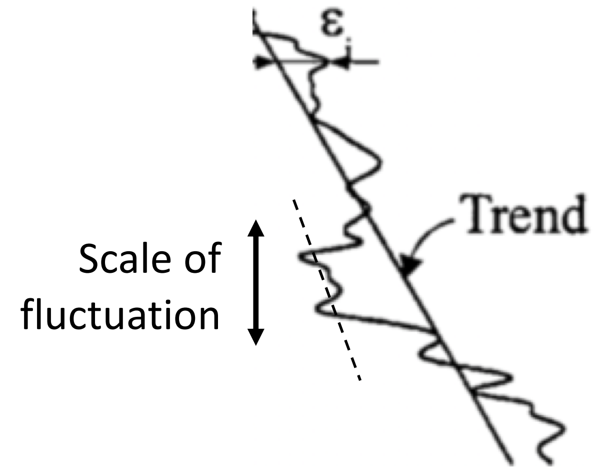
El-Ramly, 2001

- Typically, assumes stationary data average (no trend), or simple spatial trend.
- Average measure of dissimilarity between data separated by a distance (r).

$$\gamma_x(r) = \frac{1}{2} E[(x_i - x_{i+r})^2]$$

Sources of uncertainty in rock slope engineering

Parameter uncertainty



What are these telling us?

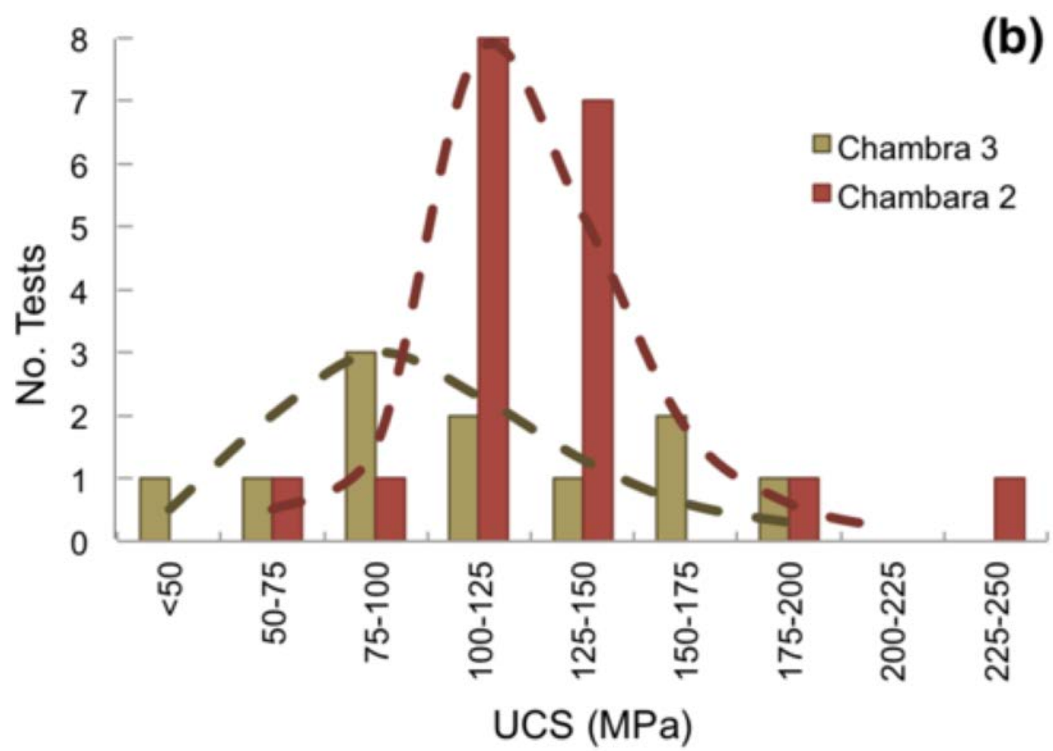
- Trends define large scale spatial variability
- *r* defines our understanding of small scale variability vs. random variability
- These values are estimated based on observations (tests) through simple trend analyses (e.g. least squares), estimates of mean and variance (method of moments) for errors around the trend.
- Take advantage of CPT for soil-type behavior
- Do we have the information for these analyses ref. geotechnical parameters in rock?
- -> We work based on geotechnical domains and we assign geotechnical parameters for each.

Sources of uncertainty in rock slope engineering

Parameter uncertainty

So now we select our domains, we go get some more stuff drilled, we have fun characterizing the discontinuities, and we get some core tested… In the example below;

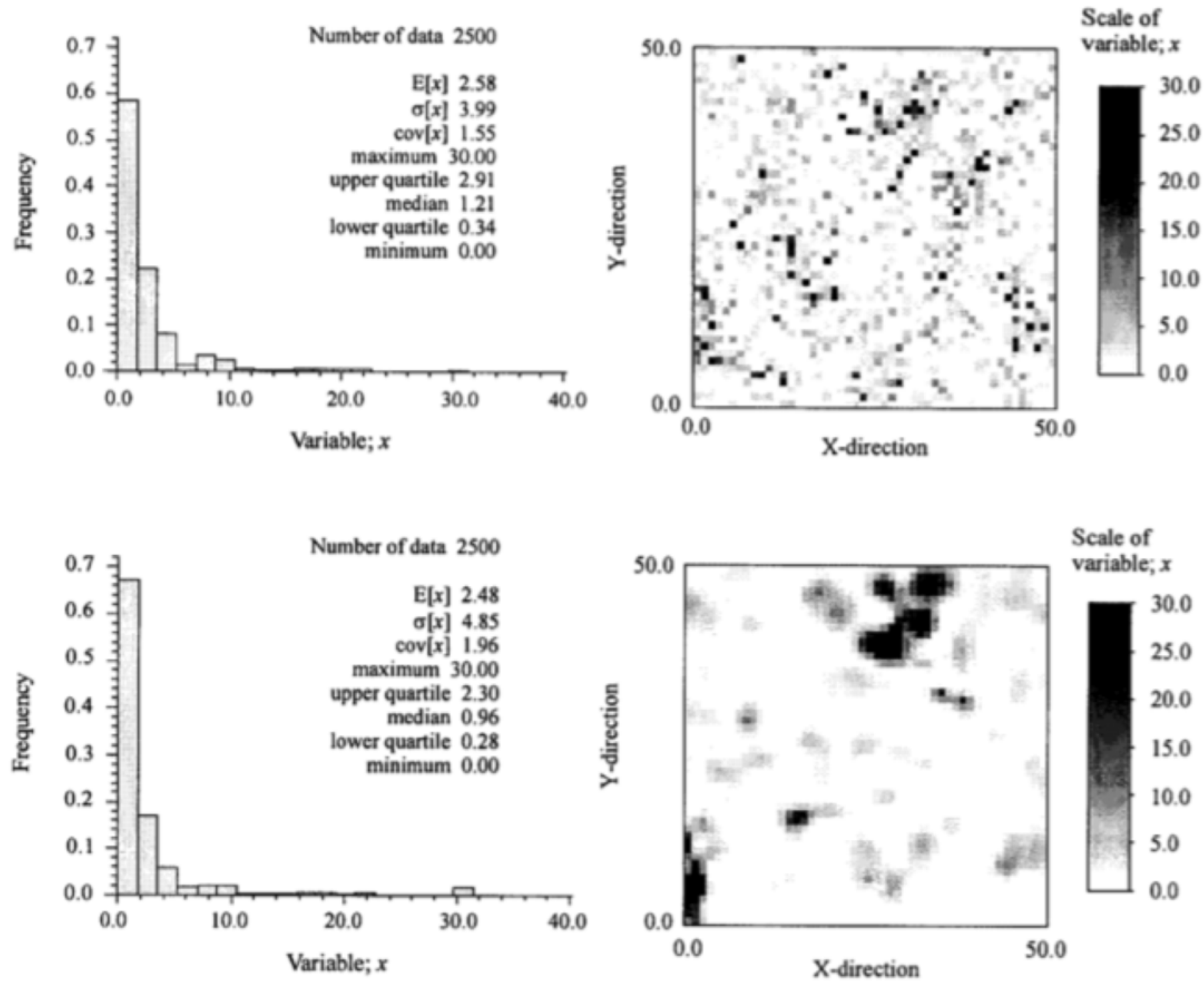
What do we use for UCS? Apply statistical techniques?



Macciotta et al. 2014

Sources of uncertainty in rock slope engineering

Parameter uncertainty



Care with blind quantification of variability without due regard for the physical reality!

Spatial variability can have a significant effect on model response, even if the same statistical characteristics are used

El-Ramly, 2001

Sources of uncertainty in rock slope engineering

Parameter uncertainty

Input of engineering judgment Required for: 1) subdividing the domain, 2) characteristic value of the parameter, 3) working range of the parameter, 4) distribution of parameter values

Example variability in basic parameters for different rocks:

Rock	σ_c (MPa)	E (GPa)	v	Rock	σ_c (MPa)	E (GPa)	v
Andesite	120–320	30–40	0.20–0.30	Granodiorite	100–200	30–70	0.15–0.30
Amphibolite	250–300	30–90	0.15–0.25	Greywacke	75–220	20–60	0.05–0.15
Anhydrite	80–130	50–85	0.20–0.35	Gypsum	10–40	15–35	0.20–0.35
Basalt	145–355	35–100	0.20–0.35	Limestone	50–245	30–65	0.25–0.35
Diabase	240–485	70–100	0.25–0.30	Marble	60–155	30–65	0.25–0.40
Diorite	180–245	25–105	0.25–0.35	Quartzite	200–460	75–90	0.10–0.15
Dolerite	200–330	30–85	0.20–0.35	Sandstone	35–215	10–60	0.10–0.45
Dolomite	85–90	44–51	0.10–0.35	Shale	35–170	5–65	0.20–0.30
Gabbro	210–280	30–65	0.10–0.20	Siltstone	35–250	25–70	0.20–0.25
Gneiss	160–200	40–60	0.20–0.30	Slate	100–180	20–80	0.15–0.35
Granite	140–230	30–75	0.10–0.25	Tuff	10–45	3–20	0.20–0.30

Source: Data selected from Jaeger & Cook (1979), Goodman (1989), Bell (2000), Gonzalez de Vallejo (2002)

Read and Stacey, 2009

- A measure of this variability is the Coefficient of Variation (COV).

$$COV = \frac{s}{\mu} \qquad \mu_x = \frac{\sum_{i=1}^{n} x_i}{n} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu_x)^2}{n-1}}$$

Sources of uncertainty in rock slope engineering

Parameter uncertainty

$$COV = \frac{s}{\mu}$$

Examples of COV

Test type	Property	Material type	Coefficient of variation (%)			
			m	mean	S.D.	range
Index	γ, γ_d	fine-grained	14	7.8	5.8	2-20
	w_n	fine-grained	40	18.1	7.9	7-46
	w_P	fine-grained	23	15.7	6.0	6-34
	w_L	fine-grained	38	18.1	7.1	7-39
	PI - all data	fine-grained	33	29.5	10.8	9-57
	- ≤ 20%	fine-grained	13	35.0	11.4	16-57
	- > 20%	fine-grained	20	26.0	9.0	9-40
	γ, γ_d	rock	42	0.9	0.7	0.1-3
	n	rock	25	25.9	19.4	3-71
Strength	ϕ', tan ϕ'	sand, clay	48	13.9	10.4	4-50
		sand	32	9.0	3.0	4-15
		clay	16	23.5	13.0	10-50
	s_u	clay	100	31.5	14.2	6-80
	q_u	rock	178	13.7	11.6	0.3-61
	$q_{t\text{-brazilian}}$	rock	74	16.6	10.4	2-58
Stiffness	$E_{t\text{-}50}$	rock	32	30.7	15.0	7-63
VST	s_u(VST)	clay	26	25.3	6.5	13-36
DMT	E_D - all data	sand	31	42.7	19.6	7-92
	- w/o outliers		30	41.1	17.6	7-69

Kulhawy et al. 2000

Sources of uncertainty in rock slope engineering

Parameter uncertainty

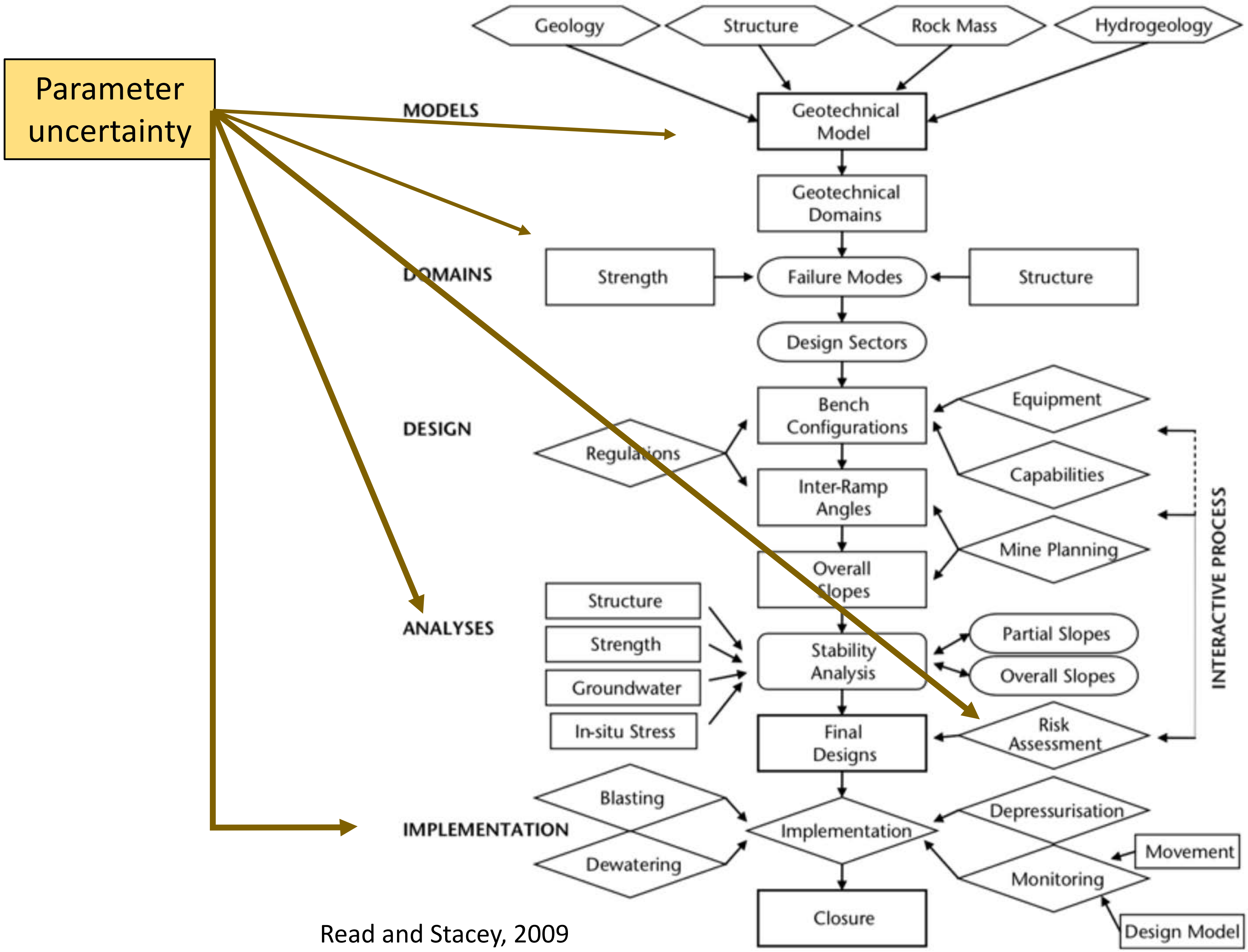
- The engineer needs to decide the approach to deal with parameter uncertainty: Characteristic values? Sensitivity analyses? Probabilistic approach? Observational method?
- These treated inherent variability – there is also bias and testing error:

point load test -> UCS -> Triaxial test -> field test

Rock mass parameters through Q, GSI, RMR

- **When does the largest impact of parameter uncertainty occur in the life of the open pit slopes?**

Sources of uncertainty in rock slope engineering



Read and Stacey, 2009

Sources of uncertainty in rock slope engineering

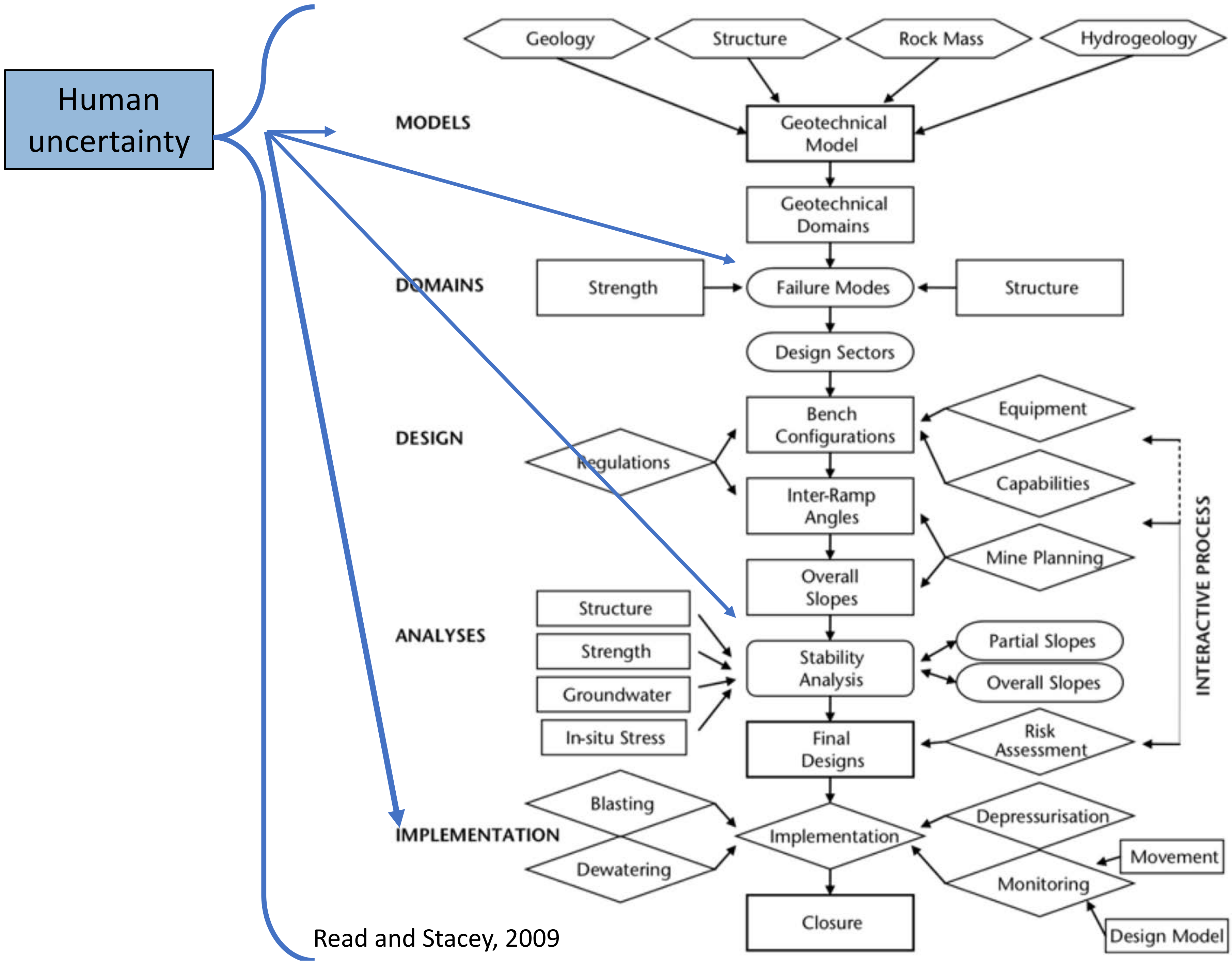
Human uncertainty

- Most difficult to address
- Skill set, work ethics, company culture, etc.
- Tools: communication, peer reviews, safety culture optimization
- Very difficult to quantify, we take a management approach through Safety Management Systems

Example Safety Management System elements (ESRM)

1) Management Leadership, Commitment and Accountability.
2) Risk Assessment and Management of Risks.
3) Community Awareness and Emergency Preparedness.
4) Management of Change.
5) Incident Reporting, Investigation, Analysis and Actions.
6) Program Evaluation (Safety Audits) and Continuous Improvement.
7) Design and Construction.
8) Operations and Maintenance.
9) Employee Competency and Training.
10) Contractor Competency and Integration.
11) Operations and Facilities Information and Documentation.

Sources of uncertainty in rock slope engineering

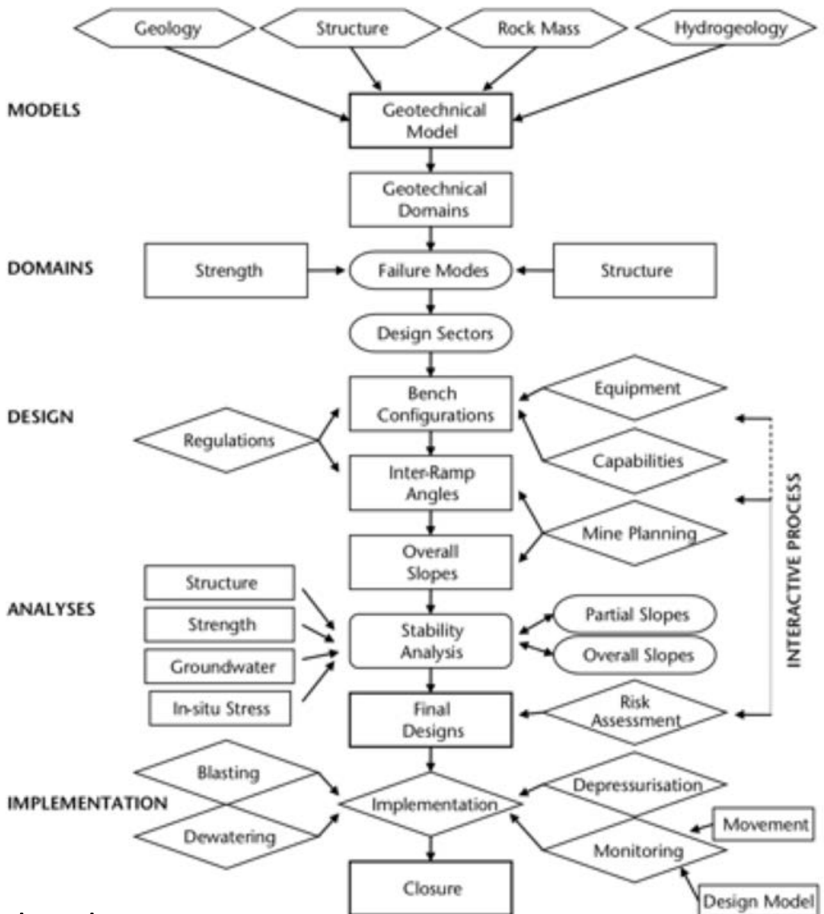


Read and Stacey, 2009

Sources of uncertainty in rock slope engineering

Human uncertainty

The trick to manage human uncertainty is to map these two into the design-construction-operation-closure



Read and Stacey, 2009

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11) Operations and Facilities Information and Documentation.

Sources of uncertainty in rock slope engineering

Can we quantify the uncertainty from all sources?

Do we need to?

Summarizing some key point the previous slides for dealing with uncertainty:

- Model uncertainty: through peer reviews, case studies
- Human uncertainty: Strong Safety Management Systems
- Parameter Uncertainty: statistical / probabilistic

This last one provides an opportunity for performance-based approaches and implementation of the Observational Method (a form of Bayesian updating)

Uncertainty and probability in slope assessments

Now you might be asking:

- How do we quantify uncertainty such that it can be propagated to reflect the likelihood of a slope failure? -> Probability of Failure (PoF)
- What is the relationship between the PoF and the Fos?
- How can slope performance influence criteria for PoF or FoS?

Lets briefly review some concepts of probability

It is not the intent to provide a primer in probability theory, but to provide a basic common understanding for our discussions.

Uncertainty and probability in slope assessments

Key notions of probability:

1) Probability is a quantitative measure of likelihood, with values between 0 (impossible) and 1 (certain).

2) Probability can measure the ratios between possible states of a system (e.g. the probability of obtaining the number 3 when tossing a die is 1/6).

3)Probability of an outcome is the number of times the outcome was observed divided by the total number of tests (frequency approach).

All apply to the evaluation of rock slopes

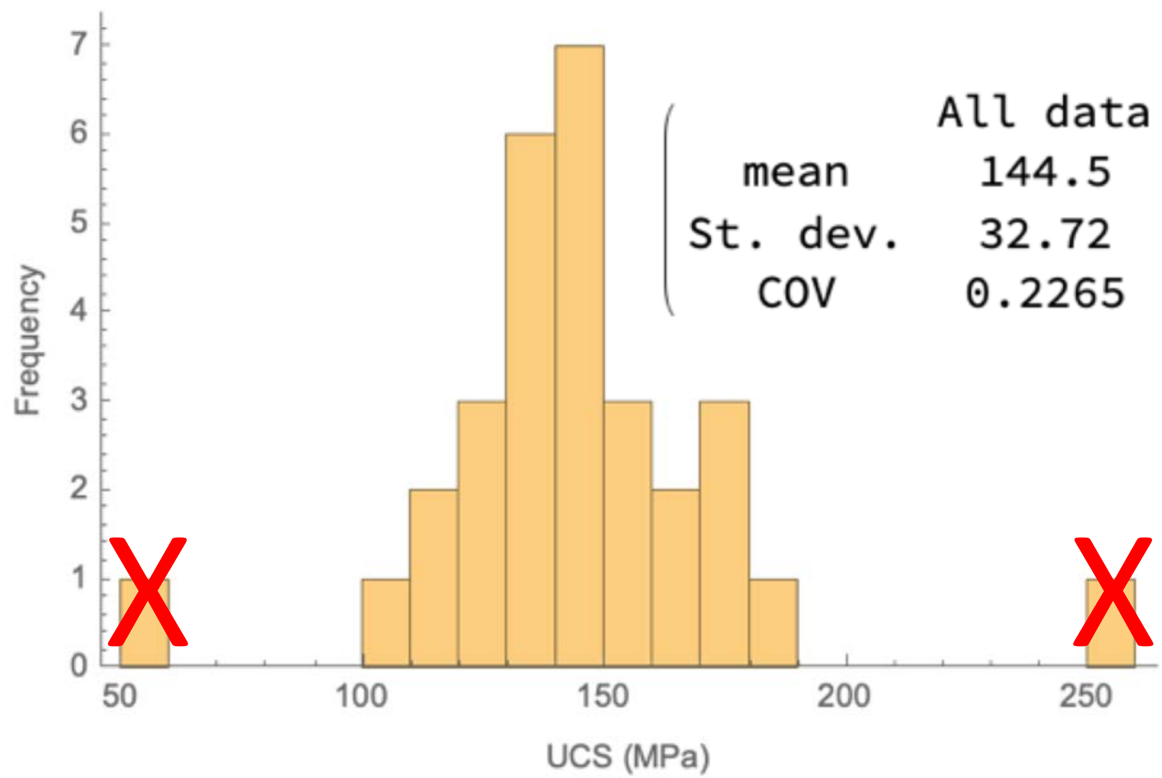
Uncertainty and probability in slope assessments

Let us use UCS results as an example:
No. tests: 30
Results (in MPa):

$$\mu_x = \frac{\sum_{i=1}^{n} x_i}{n} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu_x)^2}{n-1}}$$

$$COV = \frac{s}{\mu}$$

100	137
179	148
180	144
120	134
145	141
172	152
167	125
115	132
163	130
136	135
129	140
157	148
179	155
143	55
119	254

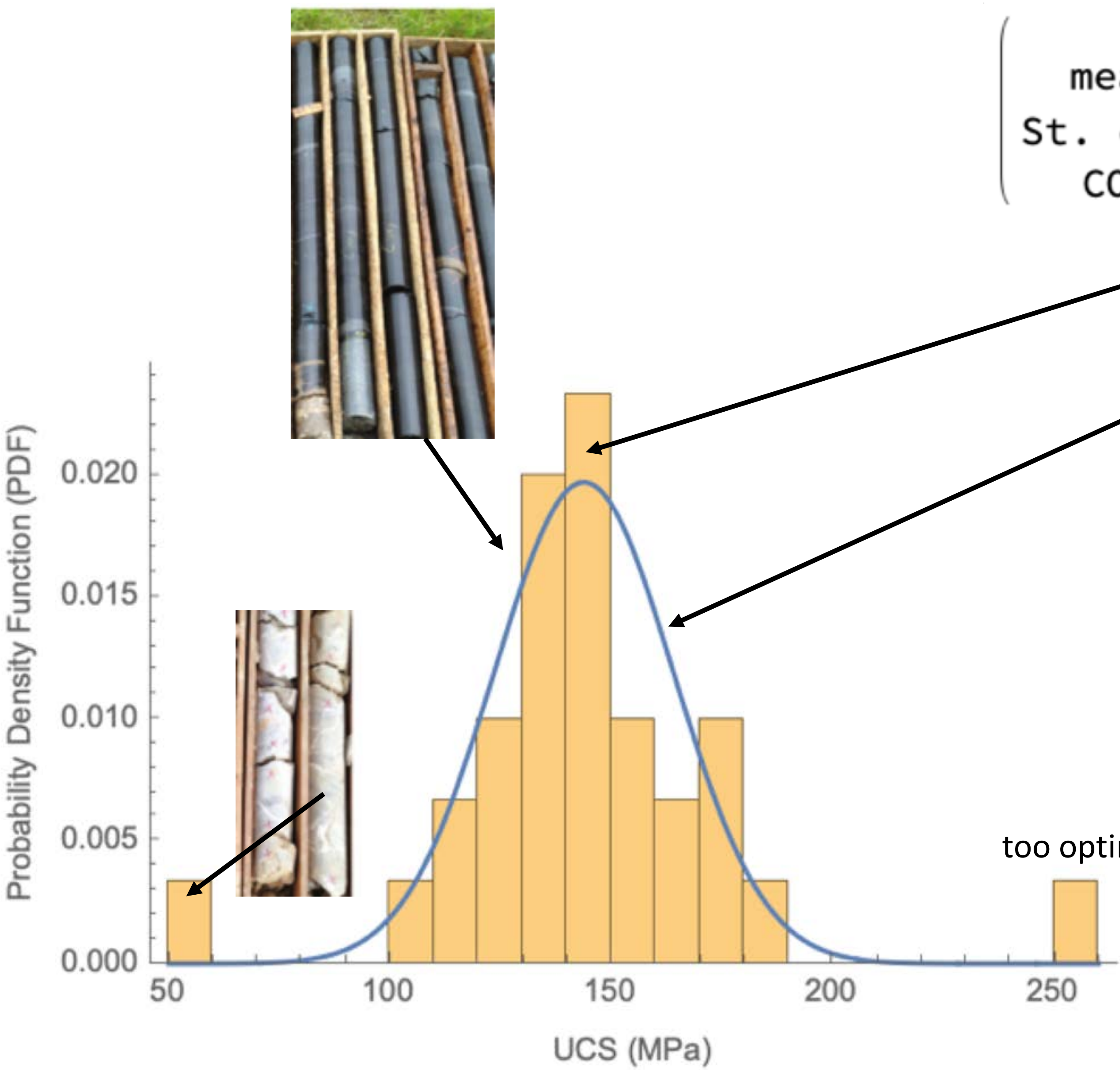


	All data	Reduced data
mean	144.5	143.8
St. dev.	32.72	20.21
COV	0.2265	0.1406

Uncertainty and probability in slope assessments

Let us use UCS results as an example:

	All data	Reduced data
mean	144.5	143.8
St. dev.	32.72	20.21
COV	0.2265	0.1406

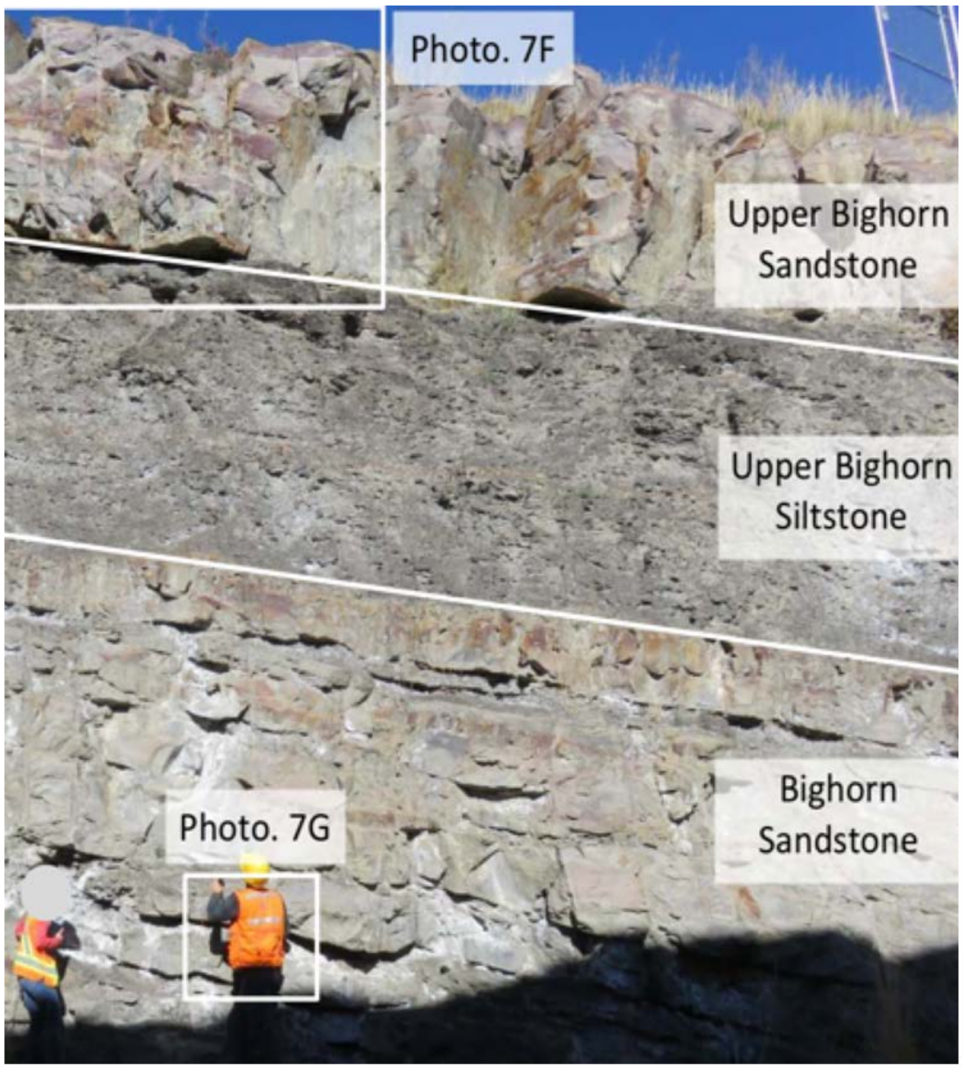


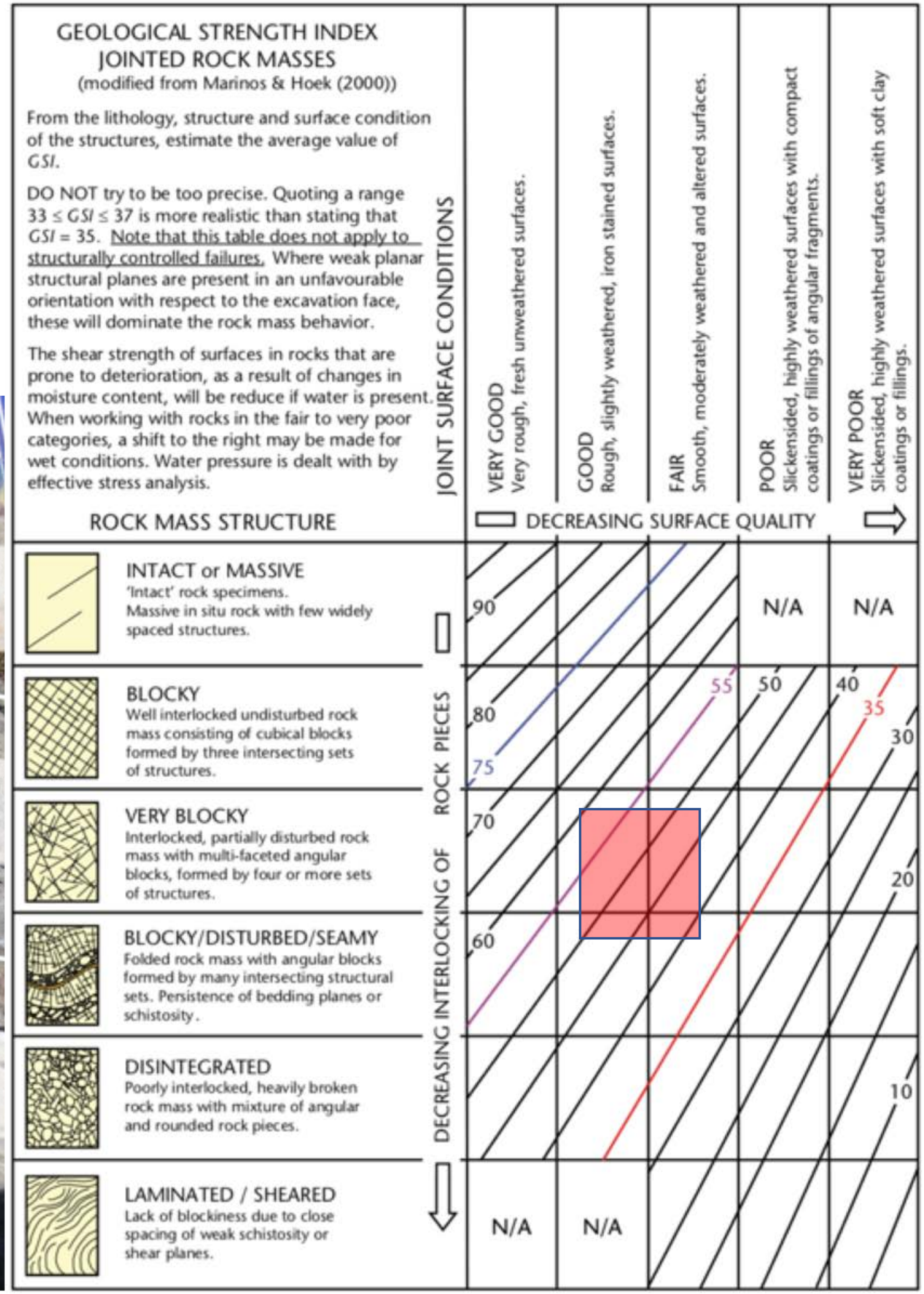
UCS Probability Density Function through frequency and judgment

What questions would you ask

Uncertainty and probability in slope assessments

Another example:
GSI for Bighorn Sandstone?

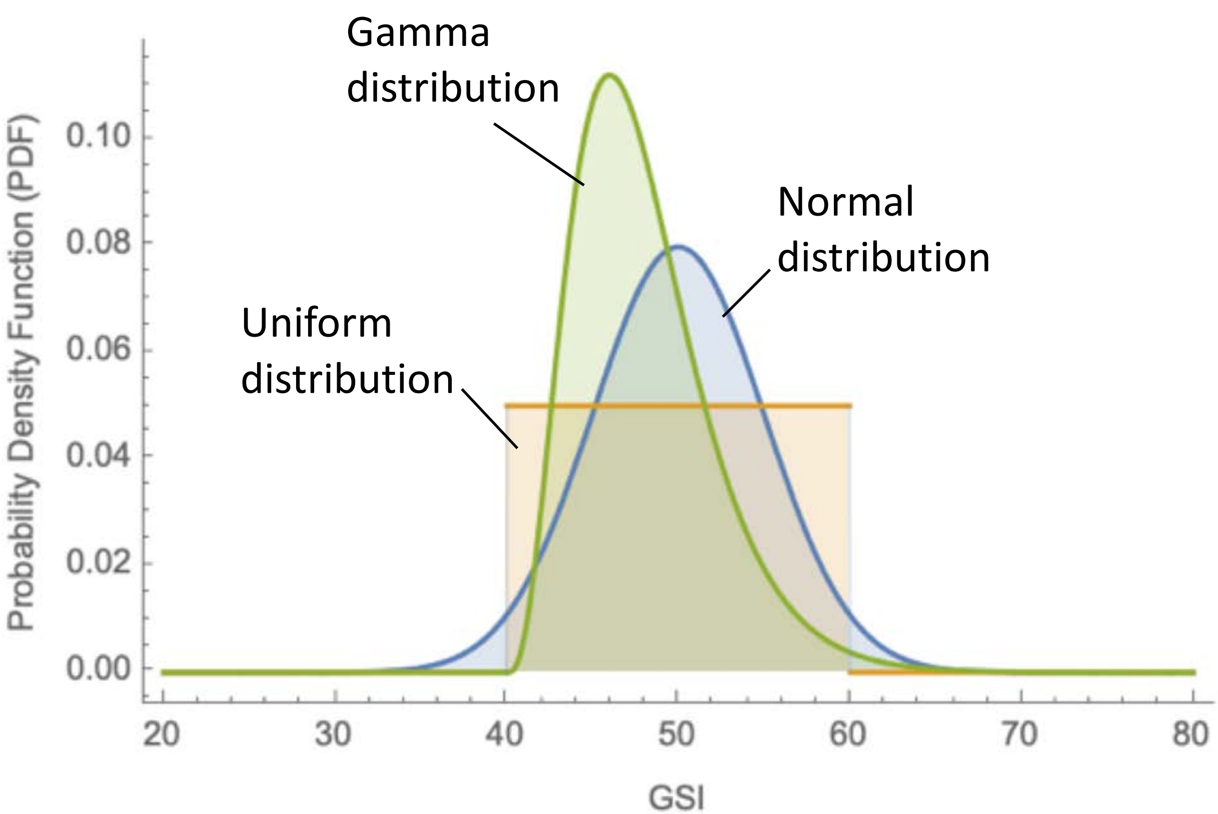




Uncertainty and probability in slope assessments

Another example:

GSI for Bighorn Sandstone, between 40 and 60?



<u>Refresher on common distributions</u>

Discrete:

- Binomial
- Bernoulli
- Poisson
- Discrete

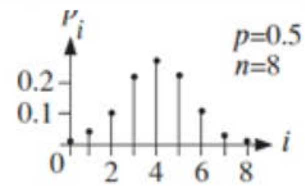
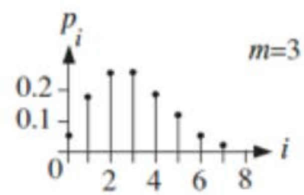
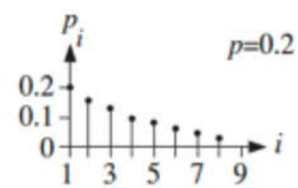
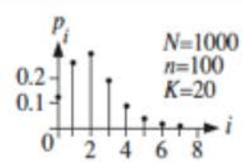
Continuous:

- Uniform
- Normal
- Triangular
- Gamma
- Log-Normal
- Exponential
- Pearson
- …
- Keep going!

Goodness of fit through visual, Q-Q plots, Chi-square tests, etc.

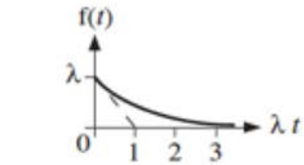
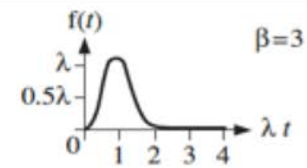
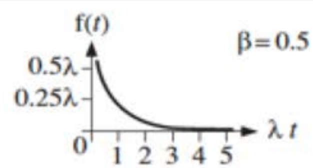
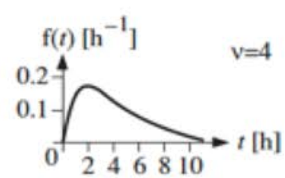
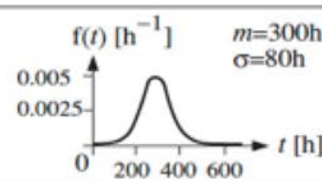
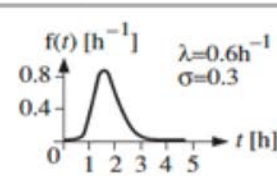
Uncertainty and probability in slope assessments

Some discrete distributions

Name	Distribution Function $F(t) = \Pr\{\tau \le t\}$	Density $f(t) = d\,F(t)/dt$	Parameter Range	Mean $E[\tau]$	Variance $Var[\tau]$
Binomial	$\Pr\{\zeta \le k\} = \sum_{i=0}^{k} p_i$ $p_i = \binom{n}{i} p^i (1-p)^{n-i}$		$k = 0, \ldots, n$ $0 < p < 1$	$n\,p$	$n\,p(1-p)$
Poisson	$\Pr\{\zeta \le k\} = \sum_{i=0}^{k} p_i$ $p_i = \frac{m^i}{i!} e^{-m}$		$k = 0, 1, \ldots$ $m > 0$	m	m
Geometric	$\Pr\{\zeta \le k\} = \sum_{i=1}^{k} p_i = 1-(1-p)^k$ $p_i = p(1-p)^{i-1}$		$k = 1, 2, \ldots$ $0 < p < 1$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hyper-geometric	$\Pr\{\zeta \le k\} = \sum_{i=0}^{k} \frac{\binom{K}{i}\binom{N-K}{n-i}}{\binom{N}{n}}$		$k = 0, 1, \ldots$ $\ldots, \min(K, n)$	$n\frac{K}{N}$	$\frac{K\,n(N-K)(N-n)}{N^2(N-1)}$

Uncertainty and probability in slope assessments

Some continuous distributions

Name	Distribution Function $F(t) = \Pr\{\tau \le t\}$	Density $f(t) = d\,F(t)/dt$	Parameter Range	Mean $E[\tau]$	Variance $Var[\tau]$
Exponential	$1 - e^{-\lambda t}$		$t > 0$ $(F(t)=0,\ t \le 0)$ $\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Weibull	$1 - e^{-(\lambda t)^{\beta}}$		$t > 0$ $(F(t)=0,\ t \le 0)$ $\lambda, \beta > 0$	$\frac{\Gamma(1+\frac{1}{\beta})}{\lambda}$	$\frac{\Gamma(1+\frac{2}{\beta}) - \Gamma^2(1+\frac{1}{\beta})}{\lambda^2}$
Gamma	$\frac{1}{\Gamma(\beta)} \int_0^{\lambda t} x^{\beta-1} e^{-x}\,dx$		$t > 0$ $(F(t)=0,\ t \le 0)$ $\lambda, \beta > 0$	$\frac{\beta}{\lambda}$	$\frac{\beta}{\lambda^2}$
Chi-square (χ^2)	$\frac{\int_0^t x^{\nu/2-1} e^{-x/2}\,dx}{2^{\nu/2}\,\Gamma(\nu/2)}$		$t > 0$ $(F(t)=0,\ t \le 0)$ $\nu = 1, 2, \ldots$ (degrees of freedom)	ν	2ν
Normal	$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} e^{-(x-m)^2/2\sigma^2}\,dx$		$\infty < t,\ m < \infty$ $\sigma > 0$	m	σ^2
Lognormal	$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln(\lambda t)}{\sigma}} e^{-x^2/2}\,dx$		$t > 0$ $(F(t)=0,\ t \le 0)$ $\lambda, \sigma > 0$	$\frac{e^{\sigma^2/2}}{\lambda}$	$\frac{e^{2\sigma^2} - e^{\sigma^2}}{\lambda^2}$

Uncertainty and probability in slope assessments

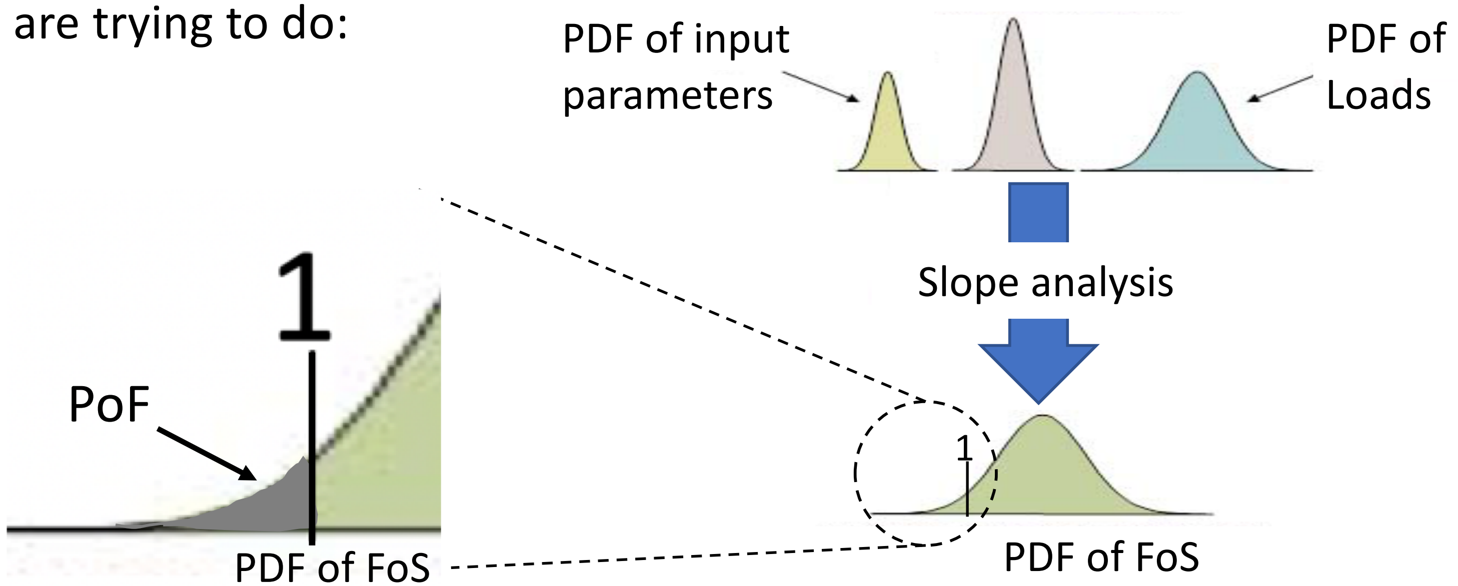
We have now a quantitative approach for parameter uncertainty within our geotechnical domains, but; how do we propagate this uncertainty and estimate a PoF?

- In a simple design for stability, we want the slope to be stable.
- That is, we want the resistance component (**R**) > load component (**Q**)

In slope design we know that a FoS $\leq$ 1 represents instability:

$FoS = R/Q \leq 1$ (Sometimes used $1 - FoS = 1 - R/Q \leq 0$)

What we are trying to do:



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Uncertainty and probability in slope assessments

How do we arrive to the distributions of R and Q, and ultimately of FoS from the parameter probability distributions?

- For simple expressions, close form solutions can provide the probability distribution of the outcome. However, even simple limit equilibrium equations are not suitable for this approach.
- Common methods for uncertainty propagation:
    - First Order Second Moment Method
    - Point Estimate Method
    - Monte Carlo-type Simulations

We will provide a brief description for reference

Uncertainty and probability in slope assessments

First Order Second Moment Method

Given that the mathematical expression for the Factor of Safety expressed as:

g(x1, x2, …xn);

First order second moment (FOSM) is based on Taylor's series expansion of a $g(x_1, x_2, \ldots x_n)$ around its mean value. For simplicity, only the linear terms of the series are usually retained. The mean and variance of the safety factor are given by

$$E[FS] = g(E[x_1], E[x_2], \ldots\ldots E[x_n])$$

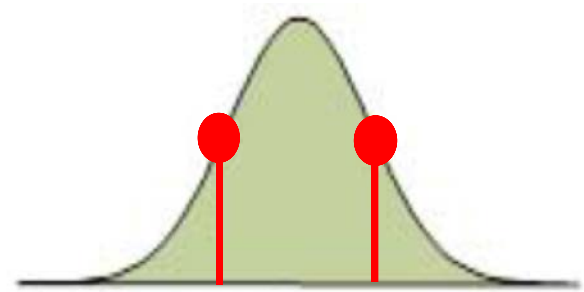
$$V[FS] \cong \sum_{i=1}^{k} \sum_{j=1}^{k} \left(\frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} \right) C(x_i, x_j)$$

where $E[FS]$ and $V[FS]$ are the mean and variance of the safety factor, respectively, and $C(x_i, x_j)$ is the covariance between input variables x_i and x_j. Detailed description of FOSM is available in many text books such as Ang and Tang (1984) and Harr (1977 and 1987).

Uncertainty and probability in slope assessments

Point Estimate Method

It is based on replacing the continuous probability distribution of input variables with discrete distributions with two values and associated probabilities such that the mean and variance of the original and discrete distributions are the same.



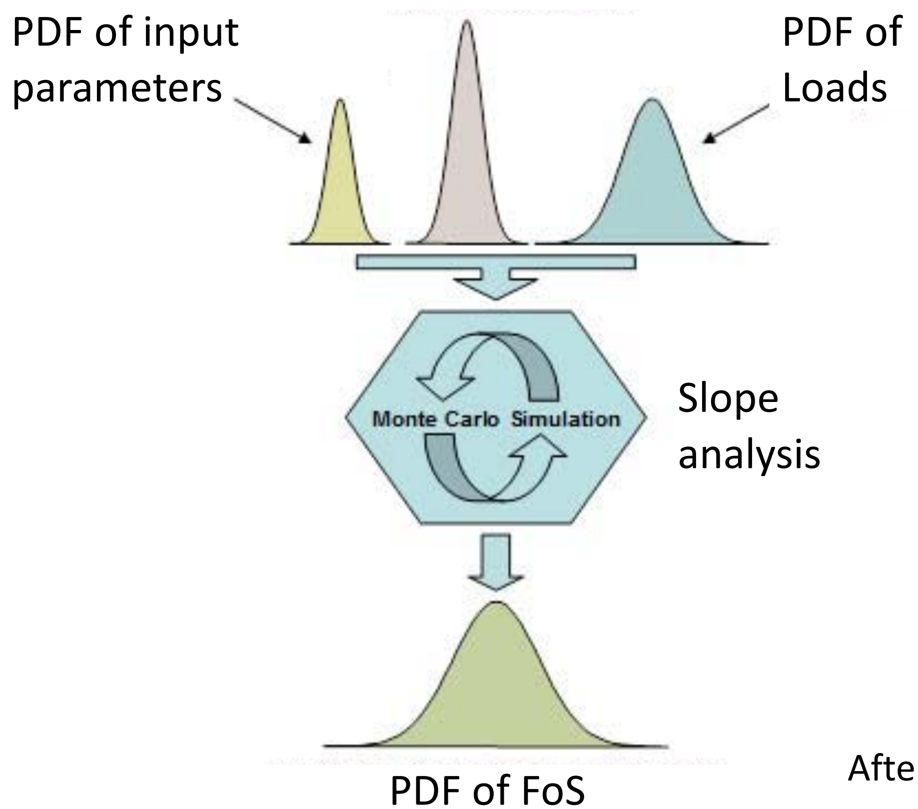
The mean and variance of the safety factor are evaluated by adding 2^n estimates of the performance function, where n is the number of input variables. These estimates constitute the values of safety factor calculated for all possible combinations of x_+ and x_- for all input variables. Commonly, x_+ and x_- are taken one standard deviation above and one standard deviation below the mean (U.S. Army, 1992). Before summing, the individual terms are multiplied by corresponding probability concentrations which are functions of correlation coefficients between variables. As the number of input variables increases, the number of terms to be evaluated increase by a power law and the analysis gets more cumbersome. The mathematical details of the technique can be found in Rosenblueth (1975, 1981) and Harr (1987). While the method is shown to be reasonably accurate for a wide range of practical problems, it can be seriously in error in some cases.

Uncertainty and probability in slope assessments

Monte Carlo Simulations – Key points

- Aims at simulating a large number of scenarios of FoS for the possible combination of parameter values, according to each parameter’s probability distribution.
- Each scenario can be a deterministic equation for the FoS or a “run” for a limit equilibrium model or SR model (although this last one with an increased amount of computation effort).

- The method selects the parameter values randomly (through random number generation). Correlation between parameters can also be added in some computational packages.
- The result is the aggregated results of FoS for all scenarios, which is treated as an observation of test results and a probability distribution of FoS.



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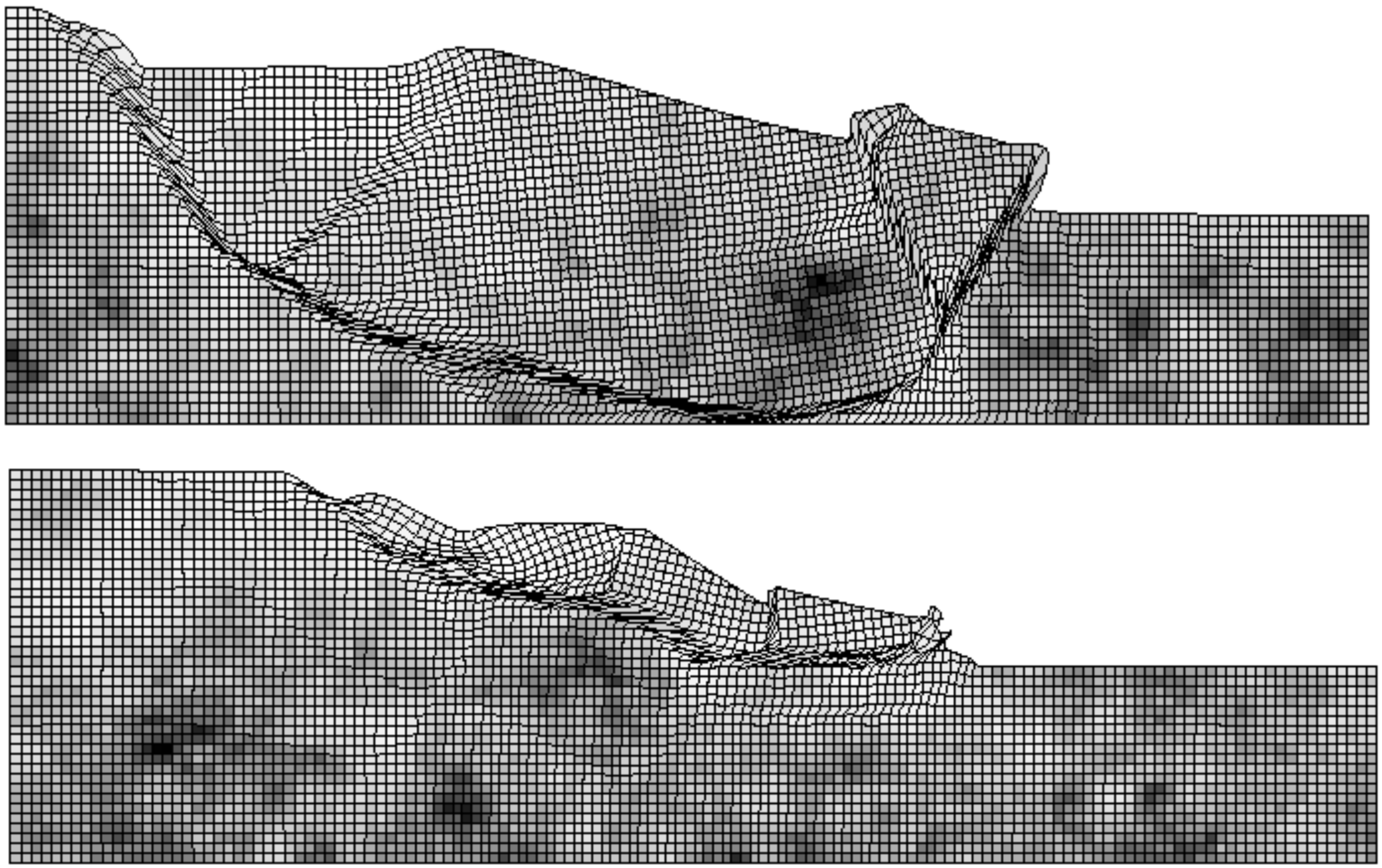
Uncertainty and probability in slope assessments

Monte Carlo Simulations – Key points

- An advantage is that it provides the full shape of the PDF, eliminating the assumptions of the shape of this PDF.
- The more complex the model, the more computational effort required
- The more parameters treated stochastically (with a PDF) the more scenarios (or iterations) are needed.

DON'T FORGET THE SPATIAL CORRELATIONS! SIGNIFICANT EFFECT ON POF!!!

Slope Stability Analysis in a Random Soil



Griffiths - 2018

Two simulations of a random slope stability analysis by RFEM. Both slopes have the same mean and standard deviation

Reliability, FoS and PoF

What is the relationship between FoS and PoF?

The "margin" for safety (**M**) can be quantified as:

$$M = R - Q$$

Where M $\leq$ 0 represents failure. If R and Q are defined by their probability distributions:

$$\mu_M = \mu_R - \mu_Q;\ \sigma_M = \sqrt{\sigma_R^2 + \sigma_Q^2 - 2\rho_{RQ}\sigma_R\sigma_Q}$$

$$\rho_{RQ} = \frac{E\left[(R-\mu_R)(Q-\mu_Q)\right]}{\sigma_R\sigma_Q}$$ *(Correlation coefficient between R and Q)*

$$\boxed{\beta = \frac{\mu_M}{\sigma_M}}$$ *(Reliability Index)*

Reliability, FoS and PoF

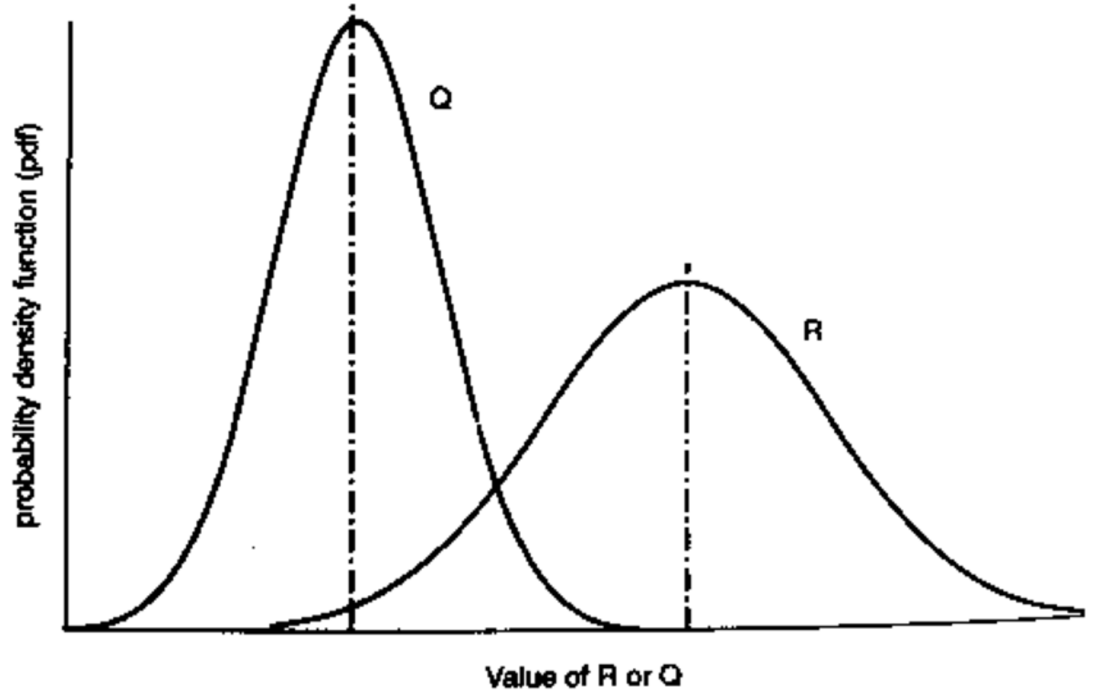
The Factor of Safety (FoS) is the common metric in slope stability analyses

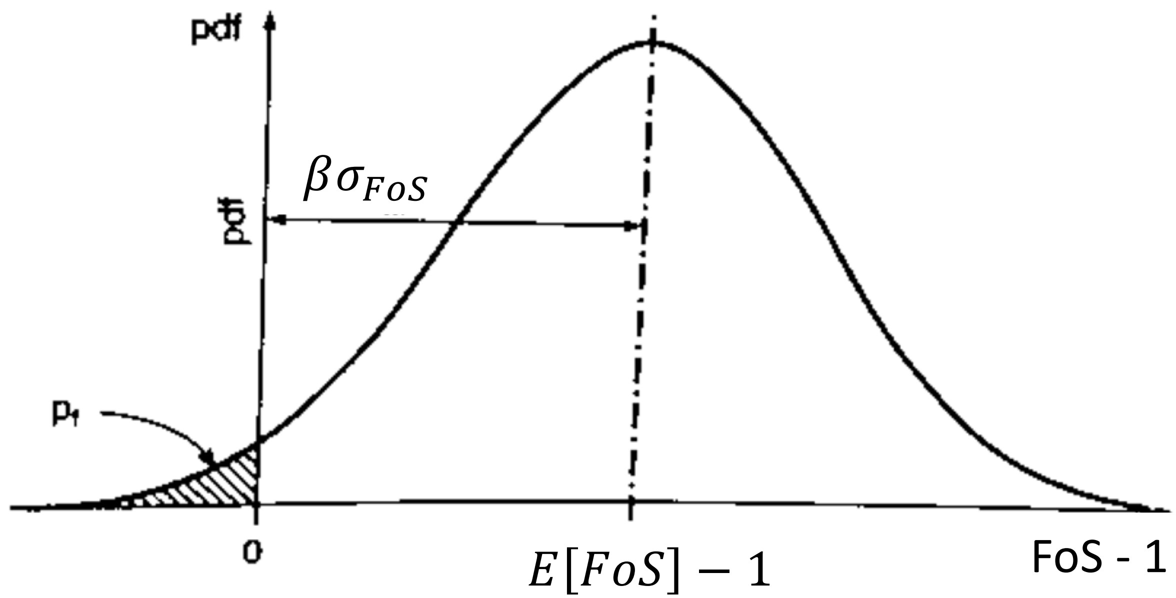
$$FoS = \frac{R}{Q}$$

and the Reliability Index:

$$\boxed{\beta = \frac{E[FoS] - 1}{\sigma_{FoS}}}$$

When using FoS, previous equations are not applicable, unless R and Q are assumed LogNormally distributed. Using the logarithms of R and Q and assuming them Normally distributed will validate the previous equations for FoS





Baecher and Christian, 2003

Reliability, FoS and PoF

This provides a means to calculate the relationship between FoS and PoF

$$\boxed{1 + \beta \times \sigma_{FoS} = E[FoS]}$$

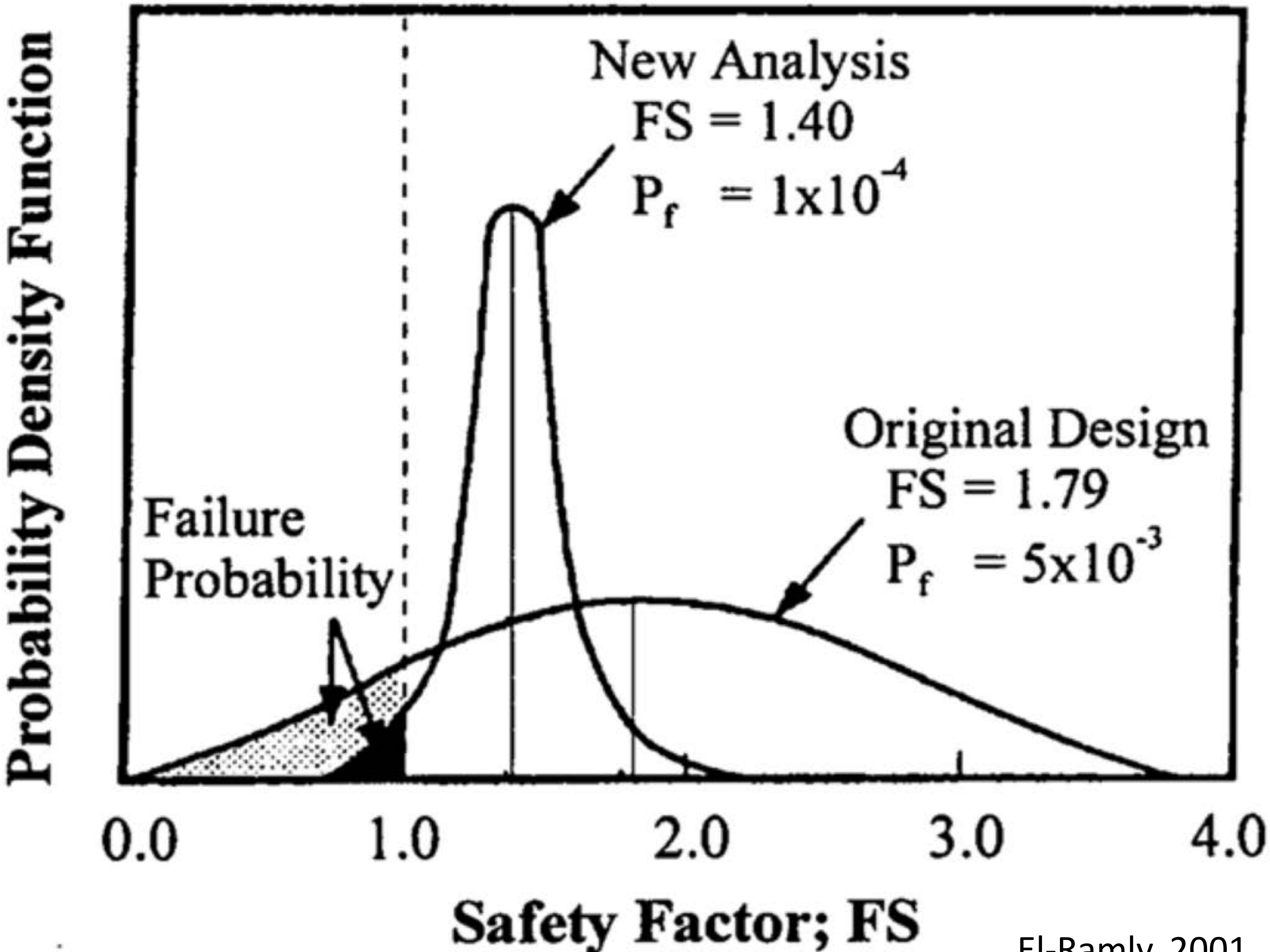
This implies that the relationship between FoS and PoF depends on the “spread” of the FoS distribution

	Probability of failure				
			LogNormal distribution		
Reliability index	Normal distribution	Triangular distribution	$\Omega = 0.05$	$\Omega = 0.10$	$\Omega = 0.15$
0.0	5.000×10^{-1}	5.000×10^{-1}	5.100×10^{-1}	5.199×10^{-1}	5.297×10^{-1}
0.5	3.085×10^{-1}	3.167×10^{-1}	3.150×10^{-1}	3.212×10^{-1}	3.271×10^{-1}
1.0	1.586×10^{-1}	1.751×10^{-1}	1.583×10^{-1}	1.571×10^{-1}	1.551×10^{-1}
1.5	6.681×10^{-2}	7.513×10^{-2}	6.236×10^{-2}	5.713×10^{-2}	5.111×10^{-2}
2.0	2.275×10^{-2}	1.684×10^{-2}	1.860×10^{-2}	1.437×10^{-2}	1.026×10^{-2}
2.5	6.210×10^{-3}	0.0	4.057×10^{-3}	2.298×10^{-3}	1.048×10^{-3}
3.0	1.350×10^{-3}	0.0	6.246×10^{-4}	2.111×10^{-4}	4.190×10^{-5}
3.5	2.326×10^{-4}	0.0	6.542×10^{-5}	9.831×10^{-6}	4.415×10^{-7}
4.0	3.167×10^{-5}	0.0	4.484×10^{-6}	1.977×10^{-7}	6.469×10^{-10}
4.5	3.398×10^{-6}	0.0	1.927×10^{-7}	1.396×10^{-9}	4.319×10^{-14}
5.0	2.867×10^{-7}	0.0	4.955×10^{-9}	2.621×10^{-12}	

$$COV(\Omega) = \frac{s}{\mu} = \frac{\sigma_{FoS}}{E[FoS] - 1}$$

Baecher and Christian, 2003

Reliability, FoS and PoF

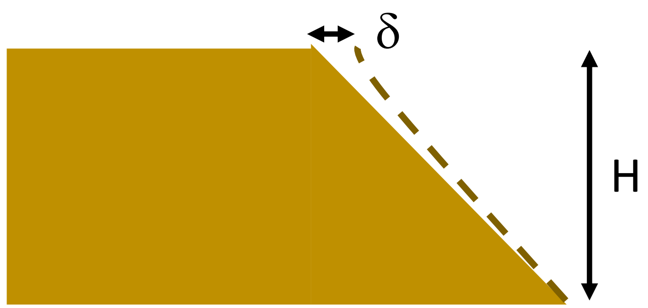


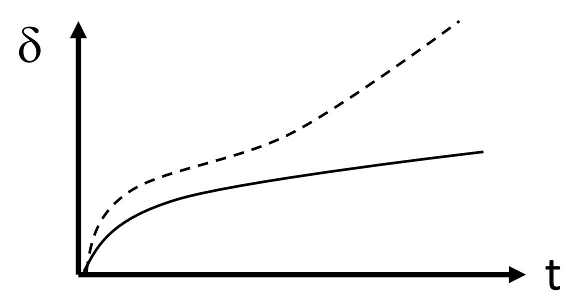
El-Ramly, 2001

This plot implies that reduction of uncertainty through enhanced field investigation, increased monitoring and back analyses, and improved modelling techniques; allows a **safer and more economic** design.

Role of monitoring and performance

- Monitoring is a very mature practice in Open Pit mining.
- This provides a continuous measure of slope performance through the life of the pit
- Measured displacements and slope deformations can be used to set criteria for slope performance, and in turn, slope geometry for future pushbacks.
- This can be as measures of slope strain (δ/H_{slope} x 100) and qualitative observations of slope deformation (elastic rebound vs. plastic deformations; constant deformation velocity vs. sustained acceleration)





These are case specific and based on site experience and will not be discussed further

Role of monitoring and performance

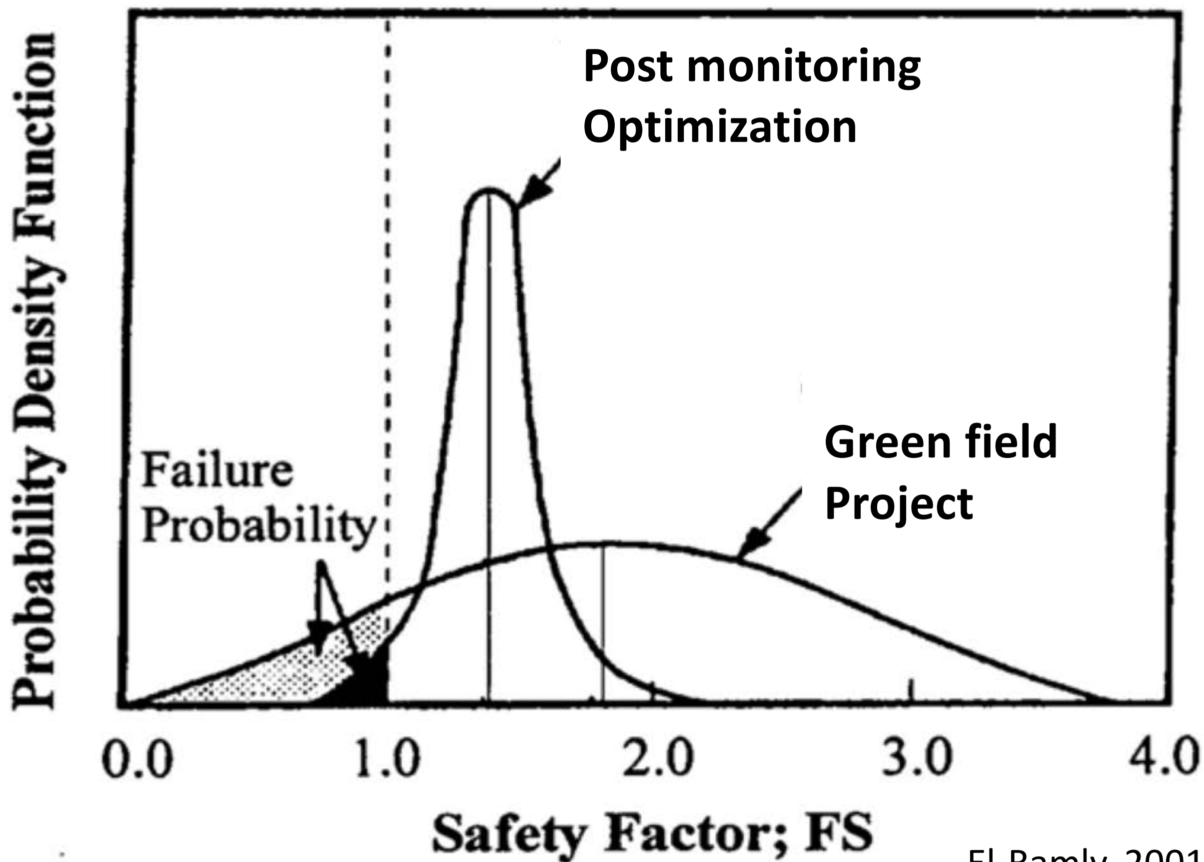
Monitoring slope performance in slope management - OM

Observational Method (OM):

1. Design under the most likely conditions. Acceptable limits of behaviour are established;
2. Worse but plausible ground conditions need to be considered to assess other potential underperformance mechanisms and those parameters that will indicate the occurrence of such scenario;
3. Design, construction, or operation modifications / enhancements are devised for the event that ground response deviates from the ranges of possible behaviour;
4. Monitoring is devised which will reveal whether the actual behaviour lies within the acceptable limits;
5. The response time of the instruments and the procedures for analysing the results shall be sufficiently rapid in relation to the possible evolution of the instability;
6. During construction and operations, the monitoring shall be carried out as planned;
7. The result of monitoring shall be assessed at appropriate stages and the planned contingency actions shall be put into operation if the limits of behaviour are exceeded;
8. Monitoring equipment shall either be replaced or extended if it fails to supply reliable data of appropriate type or in sufficient quality.

Role of monitoring and performance

What is the impact of monitoring slope performance in PoF of subsequent pushbacks?



El-Ramly, 2001

This allows a safer, more efficient design

In summary:

- Uncertainty governs most (or all) steps for rock slope design, excavation, management and closure.
- Be mindful of all potential sources of uncertainty, learn how to indent them, and know the tools available to quantify and reduce them.
- More important, know the limitations of these tools!
- Understand the design criteria. Is it based on uncertainty? Are consequences too high? Is it about deformations?
- Understand what your instrumentation is telling you. Measure and assess performance in the basis of your understanding. Do not just measure and report.
- What are your goals?

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Floor open for discussion